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# Separating the Effects of Beliefs and Attitudes on Pricing under Ambiguity

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# Separating the effects of beliefs and attitudes on pricing under ambiguity*

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**Abstract**

The pricing of an ambiguous asset, whose cash flow stream is uncertain, may be affected by three factors: the belief regarding the realization likelihood of cash flows, the subjective attitude towards risk, and the attitude towards ambiguity. While previous literature looks at the total price discount under ambiguity, this paper investigates with laboratory experiments how much effect each factor can induce. We apply both non-parametric and parametric methods to cleanly separate the belief effects, the risk premiums, and the ambiguity premiums from each other. Both methods lead to similar results: Overall, subjects have substantial ambiguity aversion, and ambiguity premiums account for the largest price deviation component when the degree of ambiguity is high. As information accumulates, ambiguity premiums decrease. We also find that beliefs do influence prices under ambiguity. This is not because beliefs are biased towards either good or bad scenarios per se, but because subjects display sticky belief updating as new information becomes available. The clear separation performed in this paper between belief and attitude also enables a more accurate estimation of the parameter of ambiguity aversion compared to previous studies, since the effect of beliefs is partialled out. Overall, we find empirically that both factors, belief and attitude towards ambiguity, are important factors in pricing under ambiguity.

**Keywords**: ambiguity, belief estimation, belief effect, ambiguity premium, laboratory experiments

**JEL code**: D81, D83

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# 1 Introduction

Ambiguity describes a situation in which some outcomes of an event have unknown/non-singular probability measurements (Ellsberg 1961; Epstein 1999; Knight 1921; Savage 1954, to name a few). Ubiquitous in financial markets, the presence of ambiguity is shown to affect individuals' behaviors such as market participation (Cao et al. 2005; Easley and O'Hara 2009), asset pricing (Bossaerts et al. 2010), and portfolio choices (Campanale 2011; Dow and da Costa Werlang 1992; Peijnenburg 2014). However, fundamental questions such as how each influencing factor determines individuals' decisions under ambiguity are not yet thoroughly studied. In particular, three factors fall into this discussion: belief, attitude towards risk, and attitude towards ambiguity. In theory, these three factors are mutually independent concepts, and thus their effects on decisions are separable (Izhakian 2020; Izhakian and Benninga 2011; Klibanoff et al. 2005). However, to the best of our knowledge, clean separation between the effect of belief and the effect of attitude is not yet achieved in empirical studies of ambiguity. Failing to disentangle beliefs from attitudes may lead to incorrect attitude characterizations and subsequently confound the estimation of the effect of attitude. Therefore, it is imperative to disentangle beliefs from attitudes and measure their effects separately. This paper aims to fill this gap.

This paper inherits the understanding that belief, attitude towards risk, and attitude towards ambiguity are three mutually separable factors which influence individuals' decisions independently. The focus of this paper is to cleanly measure the effect of each individual factor (the effects are denoted as belief effect, risk premium, and ambiguity premium, respectively.). This paper contributes to the current literature in three aspects. First, the clean measurements allow us to investigate the qualitative feature (eg. the sign) and the quantitative feature (eg. the magnitude) of each individual effect. This sheds some light on some fundamental questions in ambiguity studies: To what extent are decisions attitude-driven, and to what extent are belief-induced. Second, we identify how large the bias could be when estimating risk attitude and ambiguity attitude without isolating the effect of beliefs. This provides a view regarding how large the bias could be in the previous literature which estimates ambiguity premium or risk premium ignoring the belief effects. Third, we cleanly estimate parameters governing attitudes in the KMM model (Klibanoff et al. 2005), with belief effects being partialled out. This provides a reference of parameterization for studies conducting numerical simulations under the KMM model.

Beliefs can be defined as subjective evaluations about the ambiguous environment. It describes, in a decision maker's (DM's) opinion, how likely a possible scenario is to occur. On the other hand, attitude towards risk and attitude towards ambiguity capture a DM's preference. For instance, aversion towards risk describes that a DM prefers a less risky situation to a more risky situation. Analogously, aversion towards ambiguity describes that a DM prefers a less ambiguous situation to a more ambiguous situation. As an empirical research, this paper uses laboratory experiments to investigate beliefs, attitude towards risk, and attitude towards ambiguity of each subject participating in the experiment. In the experiments, subjects are told that there is a binary lottery whose outcome is either winning or losing. Subjects are also told that the lottery pays out a positive financial reward (eg. 3.75 Euro) in case of winning, and that the lottery pays out zero in case of losing. However, neither the winning probability nor the losing probability of the lottery is known to any subject. In this way, an ambiguous lottery is set up. To elicit beliefs, we use *guess games* to investigate a subject's belief regarding the winning probability of the ambiguous lottery. To elicit ambiguity attitude and risk attitude, we use *choice lists* to elicit a subject's certainty equivalents (CE) of the ambiguous lottery and her CEs of some purely risky lotteries, respectively. The elicited CEs carry the information of a subject's ambiguity attitude and/or risk attitude. To better analyze how subjects' beliefs and attitudes affect their decisions on CEs, we introduce an all-neutral benchmark CE which is induced by a neutral belief, risk neutrality, and ambiguity neutrality. Departure from the benchmark CE generates price deviation. According to the source of the deviation, the price deviation can be decomposed into three components: the belief effect (i.e. price deviation ascribed to a subject's personal belief deviating from the neutral benchmark belief), the risk premium (i.e. price deviation arising from a subject's non-neutral attitude towards risk), and the ambiguity premium (i.e. price deviation arising from a subject's non-neutral attitude towards ambiguity). The goal of the paper comes down to disentangling the three components of each subject's price deviation. This paper does it with two methods: a non-parametric method and a parametric method.

As a first analysis, we disentangle the three price deviation components with a non-parametric method (the results are directly derived from the experiment data). The experiment adopts a particular design: First, in a *guess game*, a subject reports her personal belief regarding the winning probability of the ambiguous lottery. Second, in a *choice list*, she reports her CE of the ambiguous lottery for which she just reports her personal belief. Third, in another *choice list*, the subject reports her CE of a purely risky lottery whose winning probability is equal to her reported personal belief. Due to this particular design (three "parallel" games in relation to one common personal belief), for each subject we can disentangle the three price deviation components, directly computed from the responses of the three "parallel" games (for details, see Chapter 3.1). To the best of our knowledge, such a clean separation of the price deviation components is not yet adopted in previous literature. This method is simple to implement and very effective for the investigation of the three deviation components. Playing these games are also cognitively easy for a subject. Such simplicity is conducive to the data quality. In addition, the non-parametric method does not require any model specification or parameter estimation, which is beneficial for the precision of this analysis. In particular, we are interested in the price deviations at two specific points in time: when no information regarding the winning probability of the ambiguous lottery is revealed (i.e. the degree of ambiguity is high), and after some information is provided but ambiguity still remains (i.e. the degree of ambiguity is comparably low).

As a second analysis, we apply a parametric method built on the KMM model. The parametric method estimates the parameters governing risk attitude and ambiguity attitude, respectively. The KMM model provides a theoretical foundation for the separation between beliefs and attitudes: risk attitude and ambiguity attitude are governed by different utility functions; Beliefs stay outside of the utility functions, and thus stay separated from attitudes. We apply the two-order probability system to formalize a subject's personal belief regarding the winning probability of the ambiguous lottery. The conceived second-order probability, which describes the likelihood that some value is the true winning probability, constitutes a subject's personal belief. In the parametric analysis, we first recover a subject's personal beliefs following the belief estimation method in Li and Wilde (2020). Subsequently, using the recovered personal beliefs as inputs, we estimate the parameter governing risk attitude and the parameter governing ambiguity attitude. Such estimations are iterated using different utility specifications: for the utility function which governs the risk attitude, a functional form representing constant relative risk aversion (CRRA) is chosen; for the utility which governs the ambiguity attitude, both functional form representing constant relative ambiguity aversion (CRAA) and functional form representing constant absolute ambiguity aversion (CAAA) are considered. The choice of the specifications refers to those in Klibanoff et al. (2005) and Izhakian and Benninga (2011). These specifications cover the main categories of utility functions applicable to the KMM model. Such variation in utility specification also provides a boarder view to understand belief effects and attitude effects under ambiguity.

With the parametric method, we further identify how large the bias could be when estimating risk attitude and ambiguity attitude without considering the effect of belief. We iterate the estimation of attitude parameters by assuming that a subject has a neutral belief. The neutral belief is characterized by a uniformly distributed initial prior and is updated following Bayes' rule. The differences between the estimated parameters based on the recovered belief and the estimated parameters based on the neutral belief capture the bias in attitude estimation if ignoring the effect of belief.

Our paper is original in the sense that there exist little ambiguity literature which empirically measures belief effect, risk premium and ambiguity premium with sound cleanness. A sizable number of literature focuses on the interplay of beliefs and attitudes instead. One stream of theoretical literature models beliefs and attitudes intertwined (Ghirardato et al. 2004; Gilboa and Schmeidler 1989; Maccheroni et al. 2006; Savage 1954; Schmeidler 1989). As a result, ambiguity premiums cannot be identified cleanly isolated from belief effects. Also in many empirical studies, beliefs are interpreted as representations of subjects' ambiguity attitudes (Abdellaoui et al. 2011; Ahn et al. 2014; Baillon et al. 2018a,b; Dimmock et al. 2016). In this literature, beliefs and attitudes are conceptually interchangeable, and thus their effects are empirically inseparable. On the other hand, some theoretical literature achieves the separation of beliefs and attitudes under ambiguity. This includes Gollier (2011), Izhakian and Benninga (2011), Izhakian (2020), Maccheroni et al. (2013), and Taboga (2005). These authors adopt the KMM model and identify effects of attitudes conditional on belief characterizations. These papers, however, do not identify belief effects explicitly. A limited number of empirical works touch the separation of beliefs and attitudes under ambiguity. Cubitt et al. (2018) elicit subjective beliefs regarding the winning probability of an ambiguous lottery as well as the subjective CEs of this lottery. The payoff of the lottery is determined by a random draw from an urn, whose composition is initially unknown to any subject. A subject's belief is elicited after a short private peek into the urn, and her ambiguity premium is estimated based on the elicited belief. The design of short private peeks indicates that the information received by each subject is idiosyncratic, and thus unobservable and uncontrollable from an experimenter's point of view. This makes the benchmark for extracting belief effect also idiosyncratic and unobservable. In our paper, all information provided to subjects are objective, without any idiosyncratic noise. This leads to an objective benchmark for extracting belief effect, and a more precise measurement of belief effects. In sum, our paper cleanly separates and measures belief effects, risk premiums and ambiguity premiums in an empirical set-up. This is original in the studies of ambiguity.

This paper reaches the following findings: (1) The belief effects on average tend to be zero when the degree of ambiguity is high, indicating that neither substantial pessimism nor substantial optimism is found in initial beliefs. As new information accumulates, belief effects emerge. This is driven by the under-adjustment of beliefs in updating compared with what Bayes' rule implies. (2) When the degree of ambiguity is high, ambiguity premiums are on average significantly positive. Overall, the ambiguity premiums are evidently larger than the risk premiums and the belief effects in a completely ambiguous environment. As new information accumulates, ambiguity premiums decrease along with the resolution of ambiguity. (3) There exists heterogeneity in all three price deviation components. The heterogeneity of ambiguity premium decreases as new information accumulates. (4) Independent from utility specification, the estimated parameter which governs ambiguity attitude implies ambiguity aversion; Ignoring the effect of belief leads to a biased estimation of risk attitude parameter and ambiguity attitude parameter, regardless of utility specification.

The rest of the paper is organized as follows: Section 2 introduces the experiment design. Section 3 derives the price deviation and its three components with a non-parametric method. Section 4 derives the price deviation and its three components with a parametric method. Section 5 concludes.

# 2 Experiment design

This paper gathers data from a series of laboratory experiments[1]. In the experiments, the ambiguous environment is operationalized by an urn. Subjects participating in the experiments are told that the urn contains in total 100 balls, and that each ball is either a white ball or a black ball. However, neither the number of white balls nor the number of black balls is known to any subject. No information is conveyed to any subject regarding how the urn is assembled. A binary lottery, whose outcome is either winning or losing, is designed based on this urn. Subjects are told that the payoff of the lottery is determined by a random draw from the urn: in case that a white ball is drawn out, the lottery pays out a positive financial reward (i.e. 3.75 Euro); in case that a black ball is drawn out, the lottery pays out zero. Therefore, an ambiguous lottery (built on the ambiguous urn) is set up. This design also implies that neither the winning probability nor the losing probability of the lottery is known to any subject.

A complete experiment session includes several parts (see Table 1). This paper uses data collected from two parts: the *guess games* which investigate subjects' personal beliefs regarding the winning probability of the ambiguous lottery, and the *choice lists* which investigate subjects' attitudes towards risk and towards ambiguity.

[1]This paper shares the experiment design and data with Li and Wilde (2020).

## 2.1 Guess games

A guess game is designed to track down a subject's personal beliefs about the composition of the ambiguous urn at some specific points in time. Equivalently, it is also to investigate a subject's conceived winning probability of the ambiguous lottery. In a guess game, a subject needs to answer the following question: standing at this point, how many white balls do you think are there in the urn? A subject enters an integer between zero and 100 (both ends are inclusive) to report her belief.

Guess games are played several times in each experiment session. Every guess game is followed by a random draw from the ambiguous urn. In each draw, one ball is drawn out from the urn and its color, either white or black, is displayed to the subjects. Then the ball is immediately put back into the urn (draw with replacement). Table 1 summarizes the sequence of the guess games and draw implementations: A subject first plays the initial guess game before any draw has occurred, denoted as $G_0$. Then the first draw is implemented. This sequence, one guess game followed by one draw implementation, repeats 15 times. In addition, in all Sessions except Session I, an additional guess game, $G_{15}$, is played after all 15 draws are observed[2]. Table 2 column (5) reports the played guess games by session. As the notation implies, when a subject plays guess game $G_n$, she has already observed $n$ times of draws ($n = 0, 1, \cdots 15$), where $n = 0$ denotes no draw is observed yet. The belief reported in guess game $G_n$ can be seen as the updated belief based on the learning of the $n$-time draw history. The past draw history, if any, is displayed on screen for subjects' reference. Figure 1, as an example, displays the screen-shot of guess game $G_5$, in which five draws are observed and this history is displayed at the bottom of the screen for subjects' reference.

Subjects participating in the same experiment session may be grouped into several markets. Table 2 column (2) reports this information. In markets 1-11, each market implements its own draws, and thus eleven independent draw history paths are generated. A subject can only observe the draw history of the market which she belongs to. For the 19 subjects in Session VII, each of them observes her own draws. This generates another 19 independent draw history paths. Table 2 column (8) summarizes this information. In total, 30 draw history paths are generated and 102 belief dynamics (one for each subject) are recorded.

The true proportion of white balls in the ambiguous urn is fixed at 40 for all sessions and for all subjects (of course, this is unknown to the subjects throughout the experiment). The choice of 40, rather than any other value, is explained in Li and Wilde (2020). Subjects are incentivized in the way that every time a subject enters the correct number of white balls (=40) in the guess game, she is rewarded with two Euro, otherwise zero. In other words, a subject is incentivized to insert the mode value of their updated belief distribution in each guess game. The incentive compatibility of this design is extensively discussed in Li and Wilde (2020). In general, eliciting the mode value of a distribution is incentive compatible (Hurley and Schogren 2005). The earning from the guess games is only announced to the subjects at the

[2]Therefore, subjects in Session I go through the sequence: Guess game $G_0$, first draw, $G_1$, second draw, $\cdots$, $G_{14}$, $15^{th}$ draw. Subjects in Session II-VII go through the same sequence, *plus* $G_{15}$ (one additional guess game after all 15 draws).

end of the experiment, hence the ambiguous feature of the urn sustains throughout the entire experiment. It is worth mentioning that the guess games only track down a subject's belief updating dynamics in the ambiguous environment, independent from her attitude towards risk and towards ambiguity. Attitude plays a role in decisions and becomes observable only when subjects actually choose between alternatives. Since the guess games do not imply any decisions between any alternatives, they are solely belief-related and independent from attitudes. Therefore, based on a subject's guess game responses, we can recover her belief dynamics and can cleanly isolate the effects ascribed to beliefs.

## 2.2 Choice lists

The choice lists compose another part of the experiment. Choice lists are designed to investigate subjects' conceived CEs of various risky lotteries, and conceived CEs of the ambiguous lottery after a certain number of draws. The design of the choice lists inherits the spirit of Becker et al. (1964) (well known as the BDM method), and resembles the paradigm in Moore and Eckel (2006), Cubitt et al. (2018) and Baillon and Placido (2019).

A subject is asked to choose between a sure positive payment (Option A) and a lottery (Option B). The lottery has a binary payoff scheme. The payout is determined by a random draw from an urn, which contains 100 balls. Each ball is either a white ball or a black ball. If a white ball is drawn out (defined as winning), the lottery pays out 1500 Experiment currency units[3] (ECU, hereafter). If a black ball is drawn out (defined as losing), the lottery pays out zero. In a choice list, a subject needs to compare 1500 different sure payments ($1, 2, \cdots, 1500$ ECU) with one specific lottery. Figure 2a presents the screenshot of a choice list. In practice, it is too tedious for a subject to submit 1500 answers in a single game. In fact, a subject is asked to report only one number, denoted as the *X-value* ($X \in \{1, 2, \cdots 1500\}$). By choosing a X-value, a subject announces that whenever the sure payment is lower than this X-value, she prefers playing the lottery; whenever the sure payment is equal to or larger than this X-value, she prefers accepting the sure payment directly. In other words, a subject's reported X-value denotes her switching point from Option B (playing the lottery) to Option A (accepting the sure payment). This paradigm already assumes the completeness and transitivity of the subjective preference. This means that a subject is guided to act rationally in a choice list in the sense that she is only allowed to switch from Option B to Option A at most once, and is never allowed to switch back to Option B again. Following the spirit of the BDM method, a subject's reported X-value is her conceived CE of the lottery.

Two kinds of choice lists are designed regarding the Option B lottery (The layout of Option A is invariant across all choice lists). In the first category, Option B is some purely risk lottery. These choice lists are prefixed with letter $R$ (as risky) and are called *risk choice lists*. In each risk choice list, a specific urn composition (the number of white balls and the number of black balls) is directly displayed on the screen, thus known to the subjects. Table 2 Column (6) summarizes the risk choice lists in each experiment session. Among them, some Option

[3]The pre-announced exchange rate is 400 ECU=1 euro. Therefore 1500 ECU=3.75 euro. The exchange rate applies to all parts of the experiment.

B lotteries are predetermined. As an example, Figure 2a displays the screen-shot of the risk choice list $R50$, where Option B is a 50-50 lottery. Analogously, $R25$ denotes the risk choice list in which Option B is a lottery urn with 25 white balls and 75 black balls. $R75$ denotes the risk choice list in which Option B is a lottery urn with 75 white balls and 25 black balls. We choose 25 and 75 as the anchors so that subjects do not encounter too extreme situations (eg. 5 white balls or 95 white balls). At the same time, this choice facilitates a rather wide coverage of lotteries, so that subjects' decisions under very different situations can be observed, conducive to the data variation. Unlike $R50/R25/R75$, Option B lotteries in $R_0/R_{15}/R_{ml}/R_{bay}$ are designed to follow a subject's reported guess game response or the draw history a subject observes. In $R_0$ ($R_{15}$), Option B is a risky lottery (urn) whose number of white balls is equal to a subject's reported guess game response in $G_0$ ($G_{15}$). Thus, $R_0$ ($R_{15}$) investigates a subject's certainty equivalent of a risky lottery with an identical white-ball proportion as the one conceived by her for the ambiguous urn before observing any draw (after observing 15 draws). Such design helps us to easily isolate a subject's ambiguity premium from the effects of belief and risk attitude (for details, see Chapter 3). In addition, some sessions also include $R_{ml}$ and $R_{bay}$ to increase data variation. In $R_{ml}$, Option B is a risky lottery whose number of white balls is equal to the maximum likelihood update after a subject observes all draws[4]. In $R_{bay}$, Option B is a risky lottery whose number of white balls is equal to the Bayesian update after a subject observes all draws[5].

For each choice list in the second category, the urn (lottery) in Option B is ambiguous. The ambiguous urn is identical to the one applied in the guess games: subjects know that the urn contains 100 balls, with either white or black color. The number of balls of either color is unknown to any subject. These choice lists are prefixed with letter $A$ (as ambiguous) and are called *ambiguity choice lists*. The ambiguity choice lists differ from each other in terms of the timing of play, i.e. how many draws are already observed. $A_n$ denotes the ambiguity choice list which is played after a subject observes $n$ draw(s) from the ambiguous urn[6]. As a example, Figure 2b displays the screen-shot of $A_5$, the ambiguity choice list played after five draws. The draw history, if any, is always displayed on the screen for subjects' reference. Table 2 Column (7) summarizes the ambiguity choice lists in each experiment session.

The incentivization method for the choice lists is as follows: after subjects finish all choice lists, as first step, the experimenters randomly select a certain number of risk/ambiguity choice lists to play for real (i.e. the earnings from these selected choice lists enter subjects' final payments.)[7]. As a second step, for each selected choice list, the experimenters randomly select one of the 1500 versions of Option A, and resolve the lottery in Option B. For each subject, she either receives the corresponding sure payment in Option A, if her reported X-value is lower than

[4]Suppose that a subject observes in total $T$ draws, with $k$ units of them being white draws, the maximum likelihood update is equal to $100k/T$.

[5]Assume that a subject starts with a uniformly distributed initial belief, and updates it in response to the draw information by employing Bayes' rule. The Bayesian update here refers to the mean value of the updated belief distribution. Chapter 4 provides more details of belief updating.

[6]$A_0$ denotes the ambiguity choice list played before any draw has occurred.

[7]The number of risk/ambiguity choice lists selected to play for real varies across experiment sessions. Table 2 Column (6)/(7) reports this information.

or equal to the value of the selected Option A, or bears the lottery result, if her X-value is larger than the value of the selected Option A. In essence, the second step is equivalent to the second step of the BDM method (randomly selecting a number $y$ to determine whether a subject takes the sure payment equal to $y$ or plays a lottery). It is well argued in Becker et al. (1964) that such paradigm is incentive compatible: a subject maximizes her profit by reporting her true certainty equivalent of the lottery in Option B. In practice, the experimenters conduct this part using some physical urns (boxes containing chips written “white” and “black”, and a box containing chips numbering $1, 2, \cdots 1500$). Random draws are executed by subjects themselves, monitored by the experimenters. Such arrangement reduces subjects’ concern about the manipulation of the urns. This procedure takes place at the very end of every experiment session, so that the ambiguous feature of the urn (in the ambiguity choice lists) sustains throughout the experiment.

## 2.3 Other information

All laboratory experiments in this paper are computerized by Z-tree (Fischbacher 2007). In total, seven sessions are conducted, with 102 participants altogether. The subjects are all randomly selected from the subject pool of the Frankfurt Laboratory for Experimental Economic Research (FLEX), Goethe University Frankfurt. Most subjects are students from Goethe University Frankfurt with various studying backgrounds, without previous experience in economics-related experiments. Table 2 Column (1)-(4) summarize the distribution of subjects in each session/market. At the beginning of each session, subjects are randomly assigned to PC terminals in the laboratory and receive printed instructions. Some quizzes, a demonstration of some physical lottery urns (in fact, boxes containing 100 chips with “white” or “black” written on), and trial draws from risky urns are implemented to help the subjects fully understand the concept of a lottery, guess games, and choice lists. The content of the ambiguous urn is not observable to any subjects, and no draws from the ambiguous urn are implemented during the demonstration. Therefore, when the guess game $G_0$ and the ambiguity choice list $A_0$ are played, the lottery urn is completely ambiguous, as intended.

Apart from the guess games and choice lists, subjects of most sessions also participate in asset trading. Table 2 Column (9) presents this information. Since the asset trading experiment is not the main interest of this paper, we suppress an extensive introduction. An experiment session concludes with a short questionnaire, followed by the final payment for each subject. Table 1 summarizes the experiment procedure. A complete experiment session lasts about 2 hours and 15 minutes on average. The total earning from all parts is on average 31 Euro.

# 3 Price deviations: non-parametric analysis

In this chapter, we extract a subject’s price deviation when pricing the ambiguous asset. In this first analysis, all price deviations are derived directly from the data of the experiments. Hence, we name it the non-parametric analysis. For each subject, we derive the total price deviation before any draw has occurred as well as after 15 draws are observed. Subsequently, we decompose the total price deviation into three components: the belief effect (i.e. price deviation ascribed to a subject's personal belief in comparison with a neutral belief), the risk premium (i.e. price deviation arising from a subject's non-neutral attitude towards risk), and the ambiguity premium (i.e. price deviation arising from a subject's non-neutral attitude towards ambiguity). Previously literature also finds price deviations (mostly price discount) when DMs price ambiguous assets. However, it is rarely clear to what extent such price deviation can be ascribed to a DM's belief characterization and to a DM's risk/ambiguity attitude, respectively. Our experiment design, which allows us to clearly disentangle the three components of the total price deviation, helps to answer these questions.

Among the many games played in a complete experiment session, a package of three "parallel" games can be used to derive the three price deviations before any draw has occurred ($n = 0$). These three games include: the guess game $G_0$ (which elicits a subject's personal belief), the risk choice list $R_0$ (which elicits a subject's certainty equivalent of a risky lottery), and the ambiguity choice list $A_0$ (which elicits a subject's certainty equivalent of the ambiguous lottery). These two choice lists, $R_0$ and $A_0$, are called "parallel" to the guess game $G_0$ since they are both associated with the personal belief elicited in $G_0$: $R_0$ constructs the risky lottery with the reported number of white balls in $G_0$; when a subject plays $A_0$, she reports her conceived certainty equivalent (CE) for exactly the ambiguous urn for which her belief has been elicited in $G_0$. Therefore, for each subject $i$, $\{G_0, R_0, A_0\}$ constitute a package, and we call it Package 0. Analogously, the three games which are played after all 15 draws are observed, $\{G_{15}, R_{15}, A_{15}\}$, constitute another package, which we call it Package 15. Using a subject's responses in Package 0 and Package 15, namely

$$\text{Responses in Package 0: } \{white_{i,0},\ X_{i,R_0},\ X_{i,A_0}\} \text{ and}$$
$$\text{Responses in Package 15: } \{white_{i,15},\ X_{i,R_{15}},\ X_{i,A_{15}}\},$$

we can directly extract a subject's total price deviation when pricing the ambiguous asset before any draw has occurred as well as after 15 draws are implemented. Furthermore, for each subject we can decompose the total price deviation into three components. The extraction of the total price deviation as well as the decomposition of the three deviation components are directly based on the raw data from the experiments, without any need for specifying a model and estimating parameters. Therefore, this straightforward method avoids any possible misspecification problem and estimation error. It can be expected to deliver high-quality results.

## 3.1 Total price deviation

First of all, we extract a subject's total price deviation when pricing the ambiguous asset. To compute the total price deviation, we first construct an all-neutral benchmark for each subject $i$. The all-neutral benchmark is the CE induced by neutral beliefs, neutral risk attitude, and neutral ambiguity attitude. Neutral beliefs imply unbiased initial beliefs (under complete ambiguity) as well as neutral belief updating (neutrally incorporating new information from the draws). Specially, we assume that benchmark belief under complete ambiguity (before any draw has occurred) is that the proportion of white balls is equal to 0.5. Given the same draw history as subject $i$ observes, the benchmark (updated) beliefs duplicate the maximum likelihood (ML) updates. Thus, we can write:

$$ML_{i,0} = 0.5 \tag{1}$$

$$ML_{i,15} = k_{i,15}/15 \tag{2}$$

$$i = 1, 2 \cdots N$$

where $k_{i,15}$ denotes the number of white draws out of 15 draws, in the draw history observed by subject $i$. The benchmark belief is assumed to be the mid-point 0.5 before any draw has occurred, since this belief neither biases towards favorable situations (with the proportion of white balls greater than 0.5) nor biases towards unfavorable situations (with the proportion of white balls less than 0.5), thus satisfying the neutrality assumption. The assumption of the ML updating is also in line with the neutrality and objectivity in belief updating. Since the benchmark is induced by risk- and ambiguity-neutrality, the benchmark CEs should be solely determined by the benchmark beliefs. In other words, $ML_{i,n}$ is equal to the all-neutral benchmark CE regarding the ambiguous lottery in ambiguity choice list $A_n$. By comparing subject $i$'s actual CE of the ambiguous asset in $A_n$ (i.e. $X_{i,A_n}$) with the benchmark CE (i.e. $ML_{i,n}$), we define the total price deviation as follow:

$$TotalDev_{i,n} = \frac{ML_{i,n} - X_{i,A_n}}{ML_{i,n}} \tag{3}$$

$$i = 1, 2 \cdots N; \quad n = 0, 15$$

All reported $X_{i,A_n}$ are standardized to $[0, 1]$. The total price deviation is a relative term, relative to the benchmark CE. We focus on the relative form to allow comparability across subjects observing different draw history paths. The results of $TotalDev_{i,n}$ are summarized in Table 3 Panel A. Figure 3 illustrates the distribution of $TotalDev_{i,n}$ for $n = 0$ and $n = 15$, respectively (the first box under each $n$ value). Overall, the total price deviation is significantly positive before any draw has occurred, and tends to be negative after 15 draws. This is supported by the results shown in Table 3. At $n = 0$, the mean value is significantly positive (mean=0.134; P-value=0.000), and the median is slightly positive (median=0.040). At $n = 15$, the mean value is negative (mean=-0.152) but less significant (P-value=0.013), and the median is only slightly away from zero (median=-0.027). These results indicate that in a completely ambiguous environment, overall subjects tend to price the ambiguous asset lower than the all-neutral benchmark, which leads to a positive total price deviation. As information accumulates, on average the total price deviation decreases and even becomes slightly negative, indicating a higher price compared with the benchmark. On an individual level, there exists large heterogeneity (Figure 3). Before any draw has occurred, the total price deviation mostly spans within the positive domain, which means that most subjects tend to price the ambiguous asset lower than the benchmark. After 15 draws, the heterogeneity sustains, but the domain shifts downwards: the total price deviation has a relatively wider coverage in the negative domain. This means that more than half of the sample have a negative total price deviation, i.e. more than half of the subjects price the asset higher than the benchmark.

As mentioned above, the total price deviation can be further decomposed into three components: belief effect, risk premium, and ambiguity premium. Defining some new variables, we can write:

$$TotalDev_{i,n} = \underbrace{BE_{i,n}}_{\text{belief effect}} + \underbrace{RP_{i,n}}_{\text{risk premium}} + \underbrace{AP_{i,n}}_{\text{ambiguity premium}} \tag{4}$$
$$i = 1, 2 \cdots N; \quad n = 0, 15$$

Analogous to $TotalDev_{i,n}$, all three components are relative terms, relative to the benchmark CE (i.e. $ML_{i,n}$). The dynamic of the total price deviation as discussed above is driven by the dynamics of the three components combined. To understand the pricing dynamics better, for each subject, we peel out each of the three components. The separation of the three components from each other can be achieved directly using the data in Package 0 and Package 15.

## 3.2 Belief effect

In this chapter, we extract the belief effects directly using subjects' responses in the guess games, the first variable in Package 0 and Package 15: $white_{i,0}$ and $white_{0,15}$. The belief effect of subject $i$ can be expressed by the difference between the benchmark belief ($ML_{i,n}$) and $i$'s personal belief ($white_{i,n}$):

$$BE_{i,n} = \frac{ML_{i,n} - white_{i,n}}{ML_{i,n}} \tag{5}$$
$$i = 1, 2 \cdots N; \quad n = 0, 15$$

where $BE_{i,n}$ denotes the belief effect of subject $i$ after observing $n$ draws, relative to the benchmark. All guess game responses, $white_{i,n}$, are expressed by the proportion of the white balls (in percent, inside interval $[0, 1]$). A positive $BE_{i,n}$ indicates that subject $i$ prices the asset lower than the benchmark belief implies, as she holds a more unfavorable belief than the benchmark belief. A negative $BE_{i,n}$ indicates that subject $i$ prices the asset higher than the benchmark belief implies, as she holds a more favorable belief than the benchmark belief. It is noteworthy that since guess games only elicit beliefs, the belief effects computed from guess game responses are thus only belief-relevant and independent from attitudes.

The belief effect can be also understood as the difference between two CEs of the ambiguous asset: the benchmark CE based on the all-neutral belief ($ML_{i,n}$) and the subject $i$'s CE based on the personal belief ($white_{i,n}$), *assuming that* $i$ is risk- and ambiguity-neutral. Given that the ambiguous lottery follows a binary Bernoulli distribution, a CE based on a specific conceived winning probability is simply the conceived winning probability itself (i.e. CE is equal to the belief in computation). From this point of view, it is also clear that belief effects are only belief-relevant and independent from attitudes, since all attitudes are assumed to be neutral.

The results of $BE_{i,n}$ are summarized in Table 3 Panel A. Figure 3 illustrates the distribution of $BE_{i,n}$ for $n = 0$ and $n = 15$, respectively (the second box under each $n$ value). On average, the personal beliefs induce little effect on asst pricing both before any draw has occurred and after 15 draws. This is supported by the fact that on average the belief effect is relative small: before any draw has occurred, the average belief effect is 0.007 but insignificant; After 15 draws, it turns to be slightly negative, but nearly insignificant (mean=-0.083, P-value=0.086).

Overall, the belief effect tends to be zero before and after 15 draws. On an individual level, however, heterogeneity prevails. As can be seen Figure 3 (the second box under each $n$ value), before any draw has occurred, there are more subjects who display price discount due to the personal beliefs ($BE_{i,n} > 0$) than subjects who display price addition ($BE_{i,n} < 0$). After 15 draws, the proportion of positive belief effects and the proportion of negative ones are nearly the same. From an individual's point of view, there seems to exist a tendency in terms of how personal beliefs influence asst pricing: a subject is more likely to price the asset lower than the benchmark belief implies, before any information is revealed, but more likely to price the asset as the benchmark belief implies, after observing a certain amount of information. It also means that a subject may bias towards the unfavorable situations in belief under completely ambiguity, and may turn to evaluating the situations neutrally as more and more information is revealed. The heterogeneity of the belief effect raises the necessity of considering the effect of belief in settings involving strong ambiguity. Without such consideration, belief effects may confound other components of the price deviation, and result in biased estimations of risk premium and ambiguity premium.

A subject's belief updating dynamic, in comparison with the benchmark belief updates (ML), can be disentangled into two inducements: the stickiness to a subject's previous belief, and the pessimism/optimism factor. Hence, the belief effect can be further decomposed into the effect of personal stickiness and the effect of pessimism/optimism. As an add-on to the discussion of belief effects, we discuss this later.

### 3.3 Risk premium

Furthermore, we peel out the risk premium, i.e. the price deviation arising from a subject's non-neutral risk attitude. We use the first two variables in Package 0, $white_{i,0}$ and $X_{i,R_0}$, to derive subject $i$'s risk premium when pricing the ambiguous asset before any draw has occurred. Recall that the risky lottery in $R_0$ has a winning probability equal to $white_{i,0}$ (i.e. subject $i$'s reported personal belief). The difference between the expected value of the lottery in $R_0$ (equal to $white_{i,0}$) and $i$'s CE of the risky lottery (i.e. $X_{i,R_0}$) captures the risk premium required by $i$. The same logic applies to $white_{i,15}$ and $X_{i,R_{15}}$ in Package 15 as well. Thus, we define:

$$RP_{i,n} = \frac{white_{i,n} - X_{i,R_n}}{ML_{i,n}} \tag{6}$$

$$i = 1, 2 \cdots N; \quad n = 0, 15$$

where $RP_{i,n}$ denotes the risk premium required by subject $i$ when pricing the ambiguous asset after observing $n$ draws. Again, $RP_{i,n}$ is a relative term, relative to the benchmark CE. All reported $X_{i,A_n}$ are standardized to $[0, 1]$. It noteworthy that the risk premium is solely risk attitude relevant: since $X_{i,R_n}$ induced by the wining probability exactly equal to $white_{i,n}$, the subtraction partials out belief effects; In addition, since the constructed lottery is purely risky, ambiguity attitude plays no role here.

A positive risk premium indicates risk aversion, a negative premium indicates risk loving, and zero indicates risk neutrality. The higher the value is, the more risk averse is subject $i$. The results of $RP_{i,n}$ are summarized in Table 3 Panel A. Figure 3 illustrates the distribution of $RP_{i,n}$ for $n = 0$ and $n = 15$, respectively (the third box under each $n$ value). Overall, risk premium tends to zero before and after 15 draws. This is supported by the fact that both mean values are close to zero and insignificant. This result indicates that, on average subjects display risk neutrality independent of draw implementation. On an individual level, however, there exists rather large heterogeneity. As can be seen in Figure 3 (the third box under each $n$ value), before any draw has occurred, slightly more subjects tend to have a positive risk premium (indicating risk aversion) than a negative risk premium (indicating risk loving). After 15 draws, there are nearly as many subjects who have positive risk premiums (N=39) as subjects who have negative risk premiums (N=41). Yet the heterogeneity intensifies.

### 3.4 Ambiguity premium

At last, we extract the ambiguity premium, i.e the price deviation arising from non-neutral ambiguity attitude. We use the last two variables in Package 0, $X_{i,R_0}$ and $X_{i,A_0}$, to derive subject $i$'s ambiguity premium when pricing the ambiguous asset before any draw has occurred. Recall that, from subject $i$'s point of view, the lottery in $R_0$ and the lottery in $A_0$ have the same (given/conceived) winning probability, which is equal to $white_{i,0}$. The only difference is that the winning probability of the lottery in $R_0$ is given (purely risky), while the winning probability of the lottery in $A_0$ is conceived to be equal to $white_{i,0}$ from $i$'s point of view (ambiguity exists). Therefore, the difference between $X_{i,R_0}$ and $X_{i,A_0}$ solely arises from $i$'s attitude towards ambiguity, and can be defined as the ambiguity premium. The same logic applies to $R_{15}$ against $A_{15}$ in Package 15. The ambiguity premium is thus defined as follow:

$$APi, n = \frac{X_{i,R_n} - X_{i,A_n}}{ML_{i,n}}; \qquad (7)$$
$$i = 1, 2 \cdots N; \; n = 0, 15$$

where $AP_{i,n}$ denotes the ambiguity premium required by subject $i$ when pricing the ambiguous asset after observing $n$ draws. Again, $AP_{i,n}$ is a relative term, relative to the benchmark CE. Since $X_{i,R_n}$ and $X_{i,A_n}$ are induced by the identical belief and the identical risk attitude, the subtraction of the two variables partials out the price deviation ascribed to belief and risk attitude. Therefore, the ambiguity premium is a clean measurement of the price deviation arising from ambiguity attitude.

A positive ambiguity premium indicates ambiguity aversion, a negative premium indicates ambiguity loving, and zero indicates ambiguity neutrality. The higher the value is, the more ambiguity averse is subject $i$. The results are reported in Table 3 Panel A. Figure 3 illustrates the distribution of $AP_{i,n}$ for $n = 0$ and $n = 15$, respectively (the fourth box under each $n$ value). Overall, subjects have significantly positive ambiguity premiums before any draw has occurred (mean=0.107, P-value=0.001), indicating strong ambiguity aversion. The ambiguity premium decreases as draws are implemented. On average, the ambiguity premium decreases to zero after 15 draws (mean=-0.040, P-value=0.385). Provided that ambiguity attitude is an inner characteristic and is consistent over time for each subject, the provision of new information (i.e. 15 draws) tend to wash out the ambiguity premium: as more information is revealed, ambiguity gradually resolves. Subjects feel more confident about their beliefs on the urn composition, and thus require less ambiguity premium for compensation. On an individual level, heterogeneity again prevails (Figure 3). Most subjects have a positive ambiguity premium (indicating ambiguity aversion) in a completely ambiguous environment ($n = 0$). After 15 draw, the distribution move downwards evidently. It seems that negative ambiguity premiums slightly outnumber the positive ones, but the difference is small. The distribution compresses from $n = 0$ to $n = 15$, indicating that heterogeneity decreases over time. This may also e explained by the resolution of the ambiguity: when ambiguity gradually resolves, more and more subjects would require smaller ambiguity premium than before. Thus, the distribution of the ambiguity premium compresses towards zero.

It is also interesting to check the proportion of each component in the total price deviation. Table 3 Panel A shows that, before any draw has occurred, on average the ambiguity premium seems to account for the largest proportion (nearly 80%) among the three components, followed by the risk premium (15%) and the belief effect (5%). In addition, the ambiguity premium is the only significant effect among the three, and it alone contributes to the significance of the total price deviation before any draw has occurred. Hence, it is rather conclusive that a subject's ambiguity attitude dominates her pricing strategy of a completely ambiguous asset. After 15 draws, all three deviations decrease to negligible amount. From the dynamics of the three components, it is rather clear that the pricing overall displays an increasing tendency, as ambiguity gradually resolves.

In terms of heterogeneity, the belief effect and the ambiguity premium tend to display decreasing heterogeneity as the number of draw increases, while the risk premium displays increasing heterogeneity. It is understandable since the belief effect and the ambiguity premium are closely affected by the draws. As ambiguity gradually resolves, more and more subjects slide to the benchmark in belief and require less compensation for exposure to ambiguity. Thus, both distributions have the tendency compressing towards zero. A subject's risk attitude is, in theory, independent from the draws, and thus the risk premium is not systematically affected by the number of draws.

## 3.5 Inducements of the belief effects: optimism or stickiness?

As an add-on discussion of the belief effect, in this chapter we analyze the possible inducements of the belief effects derived in Chapter 3.2. The belief effect is defined as the price deviation arising from the difference between the all-neutral benchmark belief characterization and a subject's personal belief characterization. When a subject updates her current belief, two factors may determine where the updated belief locates: the stickiness to her current belief, and her pessimistic/optimistic inclination. The stickiness describes, standing at her current belief ($white_{i,n-1}$), how a subject updates to her new belief ($white_{i,n}$) by reacting to the new information (represented by the $ML_{i,n}$ updates). She can fully react to the new information by perfectly following the $ML_{i,n}$ updates, i.e. $white_{i,n} = ML_{i,n}$, she can also fully stick to her current belief by ignoring the new information, i.e. $white_{i,n} = white_{i,n-1}$, or she can end up somewhere in-between. A subject's optimistic/pessimistic inclination can be represented by a further adjustment on belief after the reaction to the new information. A downward adjustment (in the conceived number of white balls) is defined as pessimism, while an upward adjustment is defined as optimism. These two factors: stickiness to current belief and pessimism/optimism, determine a subject's belief updating dynamic, and consequently induce the belief effects. The belief updating dynamic of subject $i$ can be written as follow:

$$white_{i,n} = [(1-\psi_i^s)\cdot white_{i,n-1} + \psi_i^s \cdot ML_{i,n}]^{\psi_i^p} \tag{8}$$

where $white_{i,n}$ denotes subject $i$'s elicited belief (the conceived proportion of white balls in the ambiguous urn) in the guess game $G_n$. $white_{i,n-1}$ is its lagged variable. $ML_{i,n} = k_{i,n}/n$ denotes the maximum likelihood update after $n$ draws, where $k_{i,n}$ denotes the number of white draws in the draw history observed by subject $i$. The parameter $\psi_i^s$ governs $i$'s response to the new information represented by $ML_{i,n}$. $\psi_i^s = 1$ indicates that $i$ fully responds to the new information, while $\psi_i^s = 0$ indicates that $i$ fully sticks to her current belief, ignoring the draw information. $\psi_i^s \in (0,1)$ indicates that subject $i$ partially responds to the draw information, with some degree of stickiness to her current belief. The parameter $\psi_i^p$ captures the further belief adjustment due to subject $i$'s pessimistic/optimistic inclination. In case $[(1-\psi_i^s)\cdot white_{i,n-1} + \psi_i^s \cdot ML_{i,n}] \in (0,1)$, $\psi_i^p > 1$ implies pessimism (a downward adjustment), $\psi_i^p < 1$ implies optimism (an upward adjustment), and $\psi_i^p = 1$ implies that neither pessimism nor optimism exists (no further adjustment). Based on this belief updating model, for each subject $i$, we estimate $\psi_i^s$ and $\psi_i^p$. The regression equation reads:

$$white_{i,n} = [(1-\psi_i^s)\cdot white_{i,n-1} + \psi_i^s \cdot ML_{i,n}]^{exp(p_i)} + \epsilon_{i,n} \tag{9}$$

$$\text{where} \quad \psi_i^p \equiv exp(p_i) \tag{10}$$

$$n = 1, 2, \cdots, 15; \quad i \text{ indexes subject}$$

where $\epsilon_{i,n}$ denotes the error term. $white_{i,n}$ and $white_{i,n-1}$ are observable guess game responses, and $ML_{i,n}$ are derived from the observed draw history. We first estimate $\psi_i^s$ and $p_i$ for each subject, applying nonlinear regression. Subsequently, we recover $\hat{\psi}_i^p$ based on Equation (10). Since the intention is to scrutinize the inducements of the belief effects after 15 draws ($n = 15$), we only apply the above regression to subjects in Session II-VII (N=89). Subjects in Session I (N=13) do not play the ambiguity choice list after 15 draws ($A_{15}$), hence they are excluded from the estimation. Moreover, the belief effects before any draw has occurred should be induced solely by pessimism/optimism, since at the very beginning there is no current belief to stick to (i.e. $white_{i,-1}$ is not defined). Therefore, $BE_{i,0}$ is not to be discussed.

The parameter $\hat{\psi}_i^p$ reflects to what degree $i$'s belief effect after 15 draws ($BE_{i,15}$) is induced by her pessimistic/optimistic inclination, while $\hat{\psi}_i^s$ reflects to what degree is induced by her stickiness to her current belief. Table 3 Panel C reports the results. On average, $\hat{\psi}_i^p$ is close to one (mean=0.984, N=89). For 81 subjects (91% of the 89 subjects), $\psi_i^p$ is not significantly different from one (significance level=5%). This result implies that pessimism/optimism overall is hardly the main inducement of the belief effect at $n = 15$ (i.e. $BE_{i,15}$, mean=-0.083, P-value=0.086). On the contrary, $\hat{\psi}_i^s$ is on average evidently less than one (mean=0.357, N=89). For 79 subjects (nearly 89% of the 89 subjects), $\psi_i^s$ is significantly different from one (significance level=5%). This implies that the stickiness to current beliefs play an important role in shaping the belief effects after 15 draws. Therefore, it is rather safe to conclude that the belief effects observed after 15 draws are mainly induced by subjects' stickiness to their current beliefs. The personal pessimism/optimism is negligible in belief effects, both before any draw has occurred and after all draws have occurred.

### 3.6 Main results of the non-parametric analysis

In this non-parametric analysis, we analyze the total price deviation and its three components of each subject when pricing the ambiguous asset at two specific points in time respectively: before any draw has occurred and after 15 draws. Due to the "parallel" design within the three games in Package 0/Package 15, the experiment data directly leads to clean measurements of the belief effects, the risk premiums and the ambiguity premiums. Unlike previous literature, the clean separation between beliefs and attitudes lead to a more accurate measurements of all three price deviation components. The measurements of the three components reach the following main findings: (1) before any draw has occurred ($n = 0$), price deviations (more specifically, price discounts) are observed, as previous literature also finds. On average, the ambiguity premiums are significantly positive (indicating ambiguity aversion overall), and are larger than the belief effects and the risk premiums. The belief effects are negligible: neither pessimism nor optimism is observed in subjects' initial beliefs. (2) After 15 draws are observed, subjects on average raise the price of the ambiguous asset (price discounts shrink). The positive ambiguity premiums vanish. The belief effects, however, show up. This tends to be mainly driven by subjects' stickiness to their old beliefs (under-reaction to the new information) in belief updating, not due to pessimism or optimism. (3) Heterogeneity is observed in the total price deviation and in each of its three components. The heterogeneity in ambiguity premium decreases apparently as the number of draws increase to 15.

## 4 Price deviations: parametric analysis

In the previous chapter, we extract the total price deviation and its three components for each subject when she prices the ambiguous asset after a given number of draws. Thanks to the experiment design, the deviation variables can be directly derived from the experiment data without any model specification. This technique facilitates a clean measurement for each variable. In this chapter, we apply another method based on model specification and parameter estimation, i.e. a parametric analysis. This method also leads to a clean separation of belief effects from attitude effects on asset pricing. First, we model a subject's belief updating strategy following Li and Wilde (2020). We estimate the parameters governing belief formation and belief updating using the guess game responses, and accordingly recover the personal belief dynamic of each subject. Second, we adopt the KMM model to reconstruct a subject's pricing strategy. We consider two different specifications of the utility functions, and based on each specification we estimate the parameters governing a subject's risk attitude and ambiguity attitude using the data from the risk/ambiguity choice list. Finally, with the recovered personal belief dynamics, and the estimated attitude parameters, we compute, for each subject, her (estimated) belief effect, risk premium, and ambiguity premium at the two points in time, respectively: before any draw has occurred ($n = 0$) and after 15 draws ($n = 15$). The KMM model clearly separates the beliefs from the attitudes in the preliminaries, hence the application of this model provides a clean separation between the effects ascribed to beliefs and the effects ascribed to risk/ambiguity attitudes on ambiguous asset pricing. Compared with the non-parametric analysis (the analysis directly works on the experiment data), this parametric analysis uses more data (in total 15 guess game responses) to recover the belief characterizations. It may recover a more stable belief dynamic, and helps to increase the accuracy especially in presence of subjective errors in guess game entries. In addition, the parametric analysis does not rely on some specific experiment design, as opposed to the non-parametric analysis, which relies on the "parallel" games. Therefore, it is more generalized and can be easily applied to other empirical settings involving ambiguity.

## 4.1 Belief estimation

To model a subject's asset pricing strategy, first of all, we need to recover a subject's belief dynamics along the draws. We borrow the idea of the two-order probability from the KMM model to explicitly describe a subject's belief: the proportion of white balls in the ambiguous urn (in percent), denoted as $\theta$, can take 101 scenarios: $\theta = 0, 0.01, \cdots, 1$. Hence, $(\theta, 1-\theta)$ constitutes the first-order probability. The likelihood that some $\theta$ is the true parameter constitutes the second-order probability. A subject's belief is defined as her conceived second-order probability. Specifically, subject $i$'s belief after observing $n$ draws describes how she assigns the likelihood measurement to each scenario, after observing $n$ draws. Let $Prob(\theta|i, n)$ denote $i$'s belief after observing $n$ draws. As an example, $Prob(0.2|i, 10) = 0.4$ means that from subject $i$'s point of view, the probability that the ambiguous lottery has 20 white balls (i.e. the proportion of white balls is equal to 0.2) is equal to 0.4. The belief estimation is to estimate the entire second-order probability of each subject, after a certain number of draws, and for each scenario, i.e. $Prob(\theta|i, n)$ for each subject, each $\theta = 0, 0.01 \cdots 1$, and $n = 0, 15$[8].

We apply the belief estimation method in Li and Wilde (2020) to estimate the parameters which govern a subject's initial prior and updating rule. A brief introduction of this method is presented below.

The belief formation and updating of subject $i$ is modeled as follows: subject $i$ starts with an initial prior which can be characterized by a *beta* distribution with shape parameter bundle

[8]Since the main interest of this paper is the price deviation at $n = 0$ and at $n = 15$, we restrict our attention to these two specific points in time. The beliefs after other number of draws (e.g. $n = 1, 2 \cdots 14$) are estimable based on this model as well, but not the interest of this paper, and thus we suppress the discussion about them.

$(\alpha_{i,0}, \beta_{i,0})$. She updates her belief by applying the following rule:

$$\alpha_{i,n} = \alpha_{i,0} + \gamma_i^w \cdot k_{i,n} \tag{11}$$

$$\beta_{i,n} = \beta_{i,0} + \gamma_i^b \cdot (n - k_{i,n}) \tag{12}$$

$$k_{i,0} = 0 \tag{13}$$

$$\text{all } \alpha, \beta \text{ terms} \geq 1; \quad \gamma_i^w, \gamma_i^b \geq 0; \quad n = 1, 2 \cdots T; \quad i \in \{1, 2, \cdots N\}$$

where $(\alpha_{i,n}, \beta_{i,n})$ characterizes the (updated) belief distribution of subject $i$, after she observes $n$ draws. $k_{i,n}$ denotes the number of white draws out of $n$ draws, observed by subject $i$. When no draws are observed, naturally $k_{i,0} = 0$. $T$ denotes the maximum number of draws. The updating rule expressed by Equation (11) means that after observing $n$ draws, $i$ updates her $\alpha$ parameter by adding the number of the observed white draws ($k_{i,n}$), after multiplying a subjective multiplier ($\gamma_i^w$), to her initial prior parameter ($\alpha_{i,0}$). $\gamma_i^w = 1$ indicates that she perfectly employs Bayes' rule in updating beliefs when observing a white draw. $0 < \gamma_i^w < 1$ indicates under-reaction according to Bayes' rule when observing a white draw, while $\gamma_i^w > 1$ indicates over-reaction. $\gamma_i^w = 0$ indicates no updating when observing a white draw. The same logic is applicable to the updating rule expressed by Equation (12) with respect to the black draws. An updated belief is still characterized by a *beta* distribution, with updated shape parameter bundles $(\alpha_{i,n}, \beta_{i,n})$. For a *beta*-distribution, its shape is completely governed by the bundle $(\alpha, \ \beta)$. This paper only considers the cases with $\alpha, \ \beta \geq 1$, so that the *beta*-distributions are unimodal, except for the case $\alpha = \beta = 1$ (i.e. uniform distribution)[9].

To recover subject $i$'s entire belief distributions after each draw, it is sufficient to estimate the parameter set $\{\alpha_{i,0}, \ \beta_{i,0}, \ \gamma_i^w, \ \gamma_i^b\}$ of subject $i$. Li and Wilde (2020) derive the estimation equation:

$$\frac{white_{i,n}}{100} = M_{i,n} \left[ \frac{1 - M_{i,n}}{1 - white_{i,n}/100} \right]^{\frac{\beta_{i,0} + \gamma_i^b (n - k_{i,n}) - 1}{\alpha_{i,0} + \gamma_i^w k_{i,n} - 1}} + \epsilon_{i,n} \tag{14}$$

$$\text{where} \quad M_{i,n} \equiv \frac{\alpha_{i,0} + \gamma_i^w k_{i,n} - 1}{\alpha_{i,0} + \gamma_i^w k_{i,n} + \beta_{i,0} + \gamma_i^b (n - k_{i,n}) - 2} \tag{15}$$

and $\epsilon_{i,n}$ denotes the error term. To restrict $\alpha_{i,0}, \beta_{i,0} \geq 1$ and $\gamma_i^w, \gamma_i^b \geq 0$, the exponential function is applied:

$$\alpha_{i,0} = 1 + \exp(a_i) \tag{16}$$

$$\beta_{i,0} = 1 + \exp(b_i) \tag{17}$$

$$\gamma_i^w = \exp(r_i^w) \tag{18}$$

$$\gamma_i^b = \exp(r_i^b) \tag{19}$$

[9] In case of $\alpha = \beta = 1$, the *beta*-distribution becomes a uniform distribution. In case of $\alpha = 1, \ \beta > 1$, the distribution has a decreasing PDF representation (mode$= 0$),. In case of $\alpha > 1, \ \beta = 1$, the distribution has an increasing PDF representation (mode$= 1$). In case of $\alpha > \beta > 1$, the distribution is bell-shaped and left-skewed (mode$> 0.5$). In case of $\beta > \alpha > 1$, the distribution is bell-shaped and right-skewed (mode$< 0.5$). Whenever $\alpha = \beta$, the distribution is symmetric around 0.5. In case of $\alpha = \beta > 1$, the mode is equal to 0.5. The larger the parameters are, the more squeezed is the distribution.

We plug Equation (16)-(19) into Equation (14)(15), and estimate $\{a_i,\ b_i,\ r_i^w,\ r_i^b\}$ for each subject $i$, conducting nonlinear regressions based on Equation (14). Then, we recover $\hat{\alpha}_{i,0}$, $\hat{\beta}_{i,0}$, $\hat{\gamma}_i^w$ and $\hat{\gamma}_i^b$ based on Equation (16)-(19), respectively. The $R^2$ is, on average, equal to 0.982 (max=.999, min=.695, s.d.=0.053, N=98)[10].

With the parameter estimations, we compute $\hat{\alpha}_{i,n}$ and $\hat{\beta}_{i,n}$ for each subject at $n = 0$ and $n = 15$ based on Equation (11) and (12). The estimated belief distributions can be expressed by the probability mass function (PMF) of the 101 scenarios. With $\hat{\alpha}_{i,0}$ and $\hat{\beta}_{i,0}$ ($\hat{\alpha}_{i,15}$ and $\hat{\beta}_{i,15}$), the PMF values of the belief distribution at $n = 0$ (at $n = 15$) can be computed as follows (for readability, we write $\hat{\alpha}_{i,n}$ as $\alpha$, and $\hat{\beta}_{i,n}$ as $\beta$):

$$Prob(\alpha,\ \beta,\ \theta) = \begin{cases} \text{cdf}(\alpha,\ \beta,\ 0.005); & \text{if } x = 0 \quad (20) \\ \text{cdf}(\alpha,\ \beta,\ x + 0.005) - \text{cdf}(\alpha,\ \beta,\ x - 0.005); & \text{if } x \in \{0.01, 0.02 \cdots 0.99\} \quad (21) \\ 1 - \text{cdf}(\alpha,\ \beta,\ 0.995); & \text{if } x = 1 \quad (22) \\ 1/101; & \text{for all } \theta \text{ if } \alpha = \beta = 1 \quad (23) \end{cases}$$

where $\text{cdf}(\alpha,\ \beta,\ x)$ denotes the CDF value of the *beta* distribution with shape parameters $(\alpha,\ \beta)$ evaluated at $x$. With the estimated belief parameters and the PMF computations, we finally recover the entire initial (updated) belief distributions of each subject at $n = 0$ (at $n = 15$). Figure 4a and 4b illustrate the belief distributions of each subject at $n = 0$ and at $n = 15$, respectively. For better visualization, subjects are grouped by draw history: the subjects who observe the same draw history are reported in the same sub-graph. The same draw history induces the same benchmark beliefs (the same $ML_{i,15}$; $ML_{i,0} = 0.5$ for every $i$). In each sub-graph, the benchmark belief is added as reference.

Figure 4a shows that, for most subjects, the estimated initial beliefs are rather flat: subjects assign probabilities relatively evenly to each possible scenario. Overall, it seems that no dominating proportion of probabilities are assigned to either bad scenarios (in which winning probability is less than 0.5) or good scenarios (in which winning probability is greater than 0.5 ). On average, the estimated beliefs tend to induce a similar expected value of the lottery as the benchmark belief does, which is equal to 0.5. This result seems to be robust with the finding in belief effects that $BE_{i,0}$ is on average close to zero.

In Figure 4b, relatively more squeezed estimated beliefs can be observed. This may be due to the fact that learning evolves and thus subjects become more confident about the winning probability of the ambiguous lottery. For most estimated beliefs, the peaks are very close to the benchmark beliefs, similar to Figure 4a. However, we also observe more subjects whose estimated beliefs drastically deviate from the benchmark beliefs. For example in Subgraph 3, 6, 7, at least four subjects have a belief which is much larger than the benchmark belief $ML_{i,15}$. This may help explain why the mean of belief effects after 15 draws ($BE_{i,15}$) is slightly negative: some personal beliefs bias towards favorable situations in comparison with the benchmark belief, which drives down the average belief effect. Except for these extreme values, the updated beliefs

[10] Three subjects never update their beliefs. They can be estimated by $\hat{\alpha}_{i,0} = +\infty$, $\hat{\beta}_{i,0} = \infty$, and $\hat{\gamma}_i^w = \hat{\gamma}_i^b = 0$ such that $(\hat{\alpha}_{i,0} - 1)/(\hat{\alpha}_{i,0} + \hat{\beta}_{i,0} - 2) = white_{i,0}$. One subject has extremely large values of estimations: $\hat{\alpha}_{i,0} = \exp(541)$ and $\hat{\beta}_{i,0} = \exp(539)$. These four subjects are excluded from the computation of $R^2$.

induce similar prices of the asset as the all-neutral benchmark, hence $BE_{i,15}$ on average is not far away from zero.

## 4.2 Attitude estimation

In this chapter, we adopt the KMM model to parameterize the risk attitude and the ambiguity attitude. Using the experiment data from the risk/ambiguity choice lists and the estimated belief characterizations, we estimate these two attitude parameters for each subject.

Following the KMM model, the subjective utility of the ambiguous asset can be written as follow:

$$\mathbb{E}_{\mu}\,\phi[\mathbb{E}_{\pi}\,u(c)] \tag{24}$$

For simplicity, we suppress the subject subscript $i$ and the choice list subscript ($A_n$, $R$) temporarily. The inner part, $\mathbb{E}_{\pi}\,u(c)$, denotes a vNM expected utility of a given risky scenario (one of the 101 scenarios). The risky scenario has binary outcomes, white draw or black draw. Each outcome is associated with a financial payoff $c$ and the probability measurement of this binary event is capture by $\pi$, i.e. the first-order distribution. $\mathbb{E}_{\mu}\phi(\cdot)$ denotes the vNM expected utility across all 101 scenarios. $\mu$ denotes the realization likelihood of each scenario, i.e. the second-order distribution (the belief), whose support is $\pi$. The subjective CE of an ambiguous lottery (i.e. the $X$-value elicited from an ambiguity choice list, denoted as $X$ below) can be written as:

$$X = u^{-1}(\phi^{-1}(\mathbb{E}_{\mu}\,\phi[\mathbb{E}_{\pi}\,u(c)])) \tag{25}$$

For a risk choice list, this equation degenerates to

$$X = u^{-1}(\mathbb{E}_{\pi}\,u(c)) \tag{26}$$

Specifying $u$ as well as $\phi$, and using a subject's responses in the choice lists ($X$) as well as her belief characterizations ($\mu$), we can estimate, for each subject, the parameter governing her risk attitude ($\rho$) and the parameter governing her ambiguity attitude ($\delta$). First, we specify the utility function $u$ which governs the risk attitude as follow:

$$u(c) = 1 + c^{1-\rho}; \quad \rho < 1 \tag{27}$$

where $c$ takes values in the interval $[0, 1]$[11]. We choose this specific first-order utility functional form since it has the following good properties: (1) The value of $\rho \in (-\infty, 1)$ is monotone in risk attitude. It covers all possible risk attitude levels, from infinite risk-seeking (in case of $\rho \to \infty$) to infinite risk-aversion (in case of $\rho \to 1$), with $\rho = 0$ representing risk neutrality. (2) $u(c)$ exhibits constant relative risk aversion (CRRA): $-c \cdot u''(c)/u'(c) = \rho$. As an extension, $\rho < 0$ represents a constant relative risk seeking. This supports our assumption that attitude is consistent across time for a given subject. This also implies that risk aversion is independent

[11] The asset payout scheme is to be standardized such that in case a white draw is observed, the payoff is one, and in case of a black draw, zero. Subjects' responses in all choice lists are to be re-scaled to [0, 1].

of wealth effect, i.e. $\rho$ is not a function of $c$. The CRRA utility form is used in some previous literature adopting the KMM model. This makes our paper comparable with others. (3) $u(c)$ is standardized such that $u(0) = 1$ and $u(1) = 2$, independent of $\rho$. It is a necessary condition for the second order utility $\phi$ to have a functional representation (see Theorem 1 in Klibanoff et al. 2005). Such standardization guarantees that risk attitude only affects ambiguity premium through dispersion effect, rather than leverage effect (Izhakian and Benninga 2011). This is also a reason why we do not consider a risk utility in form of constant absolute risk aversion (CARA, a power functional form). To our knowledge, CARA does not satisfy this condition. (4) $u(c)$, as can be easily shown for all values of $\rho < 1, c > 0$, is increasing in $c$ and continuously twice-differentiable with respect to $c$. (5) $u$ has a positive co-domain given $c \geq 0$, satisfying the domain condition of $\phi$.

The utility function which governs the ambiguity attitude, $\phi : \mathbb{R}^+ \to \mathbb{R}$, maps the (non-negative) expected utility value of a risky scenario into a utility measure, taking the presence of ambiguity into account. Its curvature governs the attitude towards ambiguity, with a more concave function standing for a higher degree of ambiguity aversion. We consider two specifications of $\phi$: one takes the form of constant relative ambiguity aversion (CRAA), the other takes the form of constant absolute ambiguity aversion (CAAA). The choice of CRAA and CAAA covers the mainstream utility functional forms widely used in current literature. Together with the CRRA-form $u$ in Equation (27), we consider Specification 1 and 2 described as follows:

**Specification 1: CRRA and CRAA**. In this specification, the utility function $\phi$ takes the following form:

$$\phi(y) = \begin{cases} \dfrac{y^{1-\delta} - 1}{1-\delta}; & \text{if } \delta \neq 1 \qquad (28) \\ \\ \log(y); & \text{if } \delta = 1 \qquad (29) \end{cases}$$

$\phi$ exhibits constant relative ambiguity attitude (CRAA), governed by the parameter $\delta$: $-\frac{\phi''(y)}{\phi'(y)} \cdot y = \delta$. $\delta > 0$ indicates ambiguity aversion, $\delta = 0$ indicates ambiguity neutral, and $\delta < 0$ indicates ambiguity seeking. In addition, $\phi$ is increasing in $y$ for any positive value of $y$ ($y \geq 1$ is guaranteed by the co-domain of $u$), and continuously twice-differentiable with respect to $y$. Moreover, $\phi(y)$ expressed by Equation (28) converges to $\log(y)$ in case that $\delta \to 1$. It implies that the representation in Equation (28) seamlessly overlaps that in Equation (29).

Using the specified utilities in Equation (27) and Equation (28)/(29) to rewrite Equation (25) yields:

$$X_{i,A_n} = \begin{cases} \left\{ \left[ \sum_{\theta=0}^{1} \text{pmf}(\hat{\alpha}_{i,n}, \hat{\beta}_{i,n}, \theta) \cdot (1+\theta)^{1-\delta_i} \right]^{\frac{1}{1-\delta_i}} - 1 \right\}^{\frac{1}{1-\rho_i}}; & \text{if } \delta_i \neq 1 \qquad (30) \\ \left[ exp\left( \sum_{\theta=0}^{1} \text{pmf}(\hat{\alpha}_{i,n}, \hat{\beta}_{i,n}, \theta) \cdot \log(1+\theta) \right) - 1 \right]^{\frac{1}{1-\rho_i}}; & \text{if } \delta_i = 1 \qquad (31) \end{cases}$$

where $A_n$ covers all ambiguity choice lists played by subject $i$[12]. As for the risk choice lists,

[12] $n = 0$ for Session I; $n = 0, 15$ for Session II, III and VI; $n = 0, 1 \cdots 15$ for Session IV, V and VII. Details

in which Option B is some specific risky lottery, $X_{i,j}$ (i.e. $i$'s response in the risk choice list $j$) records $i$'s CE of the risky lottery with a known winning probability, denoted as $\pi_{i,j}$. The winning probability $\pi_{i,j}$ of some risky lottery can be interpreted as a specific case under the two-order probability system: all second-order probabilities are assigned to the scenario $\theta = \pi_{i,j}$: pmf$(\theta = \pi_{i,j}) = 1$ and pmf$(\theta \neq \pi_{i,j}) = 0$. Plugging them into Equation (30) or (31), the equation degenerates to:

$$X_{i,j} = \pi_{i,j}^{\frac{1}{1-\rho_i}} \tag{32}$$

where $j$ covers all risk choice lists played by subject $i$.[13] Therefore, we can integrate all choice lists, both risk and ambiguity ones, into one regression equation expressed by Equation (30) or (31), with Equation (32) being a special case. It is also to say, for each subject $i$, we can estimate her ambiguity attitude ($\delta_i$) and her risk attitude ($\rho_i$) simultaneously, applying nonlinear regression based on Equation (30) or (31) (after adding an error term to the equation), using all her responses in the choice lists. Finally, the estimated parameters ($\hat{\rho}_i$ and $\hat{\delta}_i$) are obtained from the estimation (based on Equation 30 or Equation 31) which achieves the better goodness of fit (i.e. the higher $R^2$). Two subjects have no estimation results: for these two subjects, $X_{i,j}$ =constant for all $j$. No variance is found in the data and thus the estimation is not applicable. For the rest 100 subjects, the estimation tends to fit the data very well: the $R^2$ is on average 0.94

Table 4 Specification 1a summarizes the results of $\hat{\rho}_i$ across subjects. On sample-population level, subjects on average tend to display risk neutrality. Although the mean value of $\hat{\rho}_i$ across 100 subjects is slightly negative (mean$=-0.522$), it is not significantly different from zero. In addition, for nearly 65% of the subjects, the null hypothesis indicating risk neutrality ($H_0 : \rho_i = 0$) cannot be rejected at 5% (not shown in the table). Moreover, there are nearly as many risk averse subjects ($N = 49$) as risk loving subjects ($N = 50$). Overall, subjects tend to display risk neutrality. This finding seems to be robust to the risk premiums found in Chapter 3.3: risk premiums are on average close to zero, indicating risk neutrality. On an individual level, there exists heterogeneity in terms of risk attitude. This is justified by the large standard deviations of $\hat{\rho}_i$.

As for the ambiguity attitude, Table 4 Specification 1a summarizes the results of $\hat{\delta}_i$ across subjects. On a sample-population level, subjects on average display ambiguity aversion. The mean value of $\hat{\delta}_i$ across the 100 subjects is over 50 and significantly differently from zero. This implies strong ambiguity aversion. In addition, the subjects with positive $\hat{\delta}_i$ values ($N = 67$) clearly outnumber the subjects with negative ones ($N = 29$). This evidence leads to apparent overall ambiguity aversion in our sample. On an individual level, there exists heterogeneity in ambiguity attitude across subjects. This can be seen from the large standard deviations of $\hat{\delta}_i$ in each domain (except for $\hat{\delta}_i = 0$). Yet, the ambiguity averse subjects obviously dominate in proportion. These results are robust to the findings of $AP_{i,0}$ in Chapter 3.4: ambiguity premiums are strongly positive, indicating ambiguity aversion. The close-to-zero average $AP_{i,15}$

can be found in Table 2 Column (7).

[13]Details can be found in Table 2 Column (6).

in Chapter 3.4 also does not contradict with the parameter estimation result: after 15 draws, the resolution of ambiguity may explain the reduction of ambiguity premium.

The estimation of attitude parameters above uses the estimated beliefs as input: $\text{pmf}(\hat{\alpha}_{i,n}, \hat{\beta}_{i,n}, \theta)$ are the estimated beliefs in form of probability masses. The heterogeneous estimated beliefs, which precisely capture each subject's real belief dynamic, seem to lead to a more accurate estimation of the attitude parameters in comparison with the case in which the heterogeneity in personal belief is ignored. To see this point more clearly, we construct an all-neutral belief dynamic for each subject $i$: she starts with a uniformly distributed initial belief, and updates the beliefs by perfectly employing Bayes' rule in response to the draw information that she observes. Based on the belief estimation model, this belief dynamic of subject $i$ can be recovered by $\alpha_{i,0} = \beta_{i,0} = \gamma_i^w = \gamma_i^b = 1$, together with $k_{i,n}$ which represents the (subject-specific) draw history. Subsequently, we re-estimate both attitude parameters based on Equation (30) and (31), pretending that the all-neutral belief dynamic is $i$'s true belief. The results of $\hat{\rho}_i$ and $\hat{\delta}_i$ based on the all-neutral belief dynamics are reported in Table 4 Specification 1b. The mean value of $\hat{\rho}_i$ assuming all-neutral beliefs is $-0.365$, different from the mean value of $\hat{\rho}_i$ using estimated beliefs (-0.522). Such different is, however, not drastically large. The mean value of the difference, reported in the "1a minus 1b" column, is rather small (-0.157) and insignificant. These results seem to indicate that if the heterogeneous personal beliefs are not correctly characterized, the estimation of the risk attitude may result in slightly biased result.

As for the ambiguity attitude, the $\hat{\delta}_i$ estimated based on the constructed all-neutral beliefs tend to be lower than the $\hat{\delta}_i$ estimated based on the estimated personal beliefs ("the correct beliefs"). The mean value of $\hat{\delta}_i$ in Specification 1b is much lower than that in Specification 1a, and it is not significantly different from zero as well. In addition, for 62 subjects, their $\hat{\delta}_i$ values tend to be underestimated under the all-neutral beliefs, much more than the subjects whose $\hat{\delta}_i$ tend to be overestimated under the all-neutral beliefs (N=33). As can be seen in column "1a minus 1b", the mean value of the different between $\hat{\delta}_i$ using estimated beliefs and $\hat{\delta}_i$ assuming all-neutral beliefs is as large as 38.507. Moreover, Figure 5a illustrates the magnitudes of the under- and over-estimations. It shows that underestimations, which are represented on the right side of the graph, tend to be more severe than the over-estimations on the left side. All in all, these results seem to imply that if the heterogeneous personal beliefs are not correctly characterized, the estimation of the ambiguity attitude would probably be downwards biased: the real magnitude of the ambiguity aversion would be underestimated under the mistakenly characterized beliefs in our sample.

Some previous literature which adopts the KMM model for asset pricing analysis also estimates the parameters governing risk attitude and ambiguity attitude. Since different specifications of the utility functions in the KMM model can lead to different interpretations of the attitude parameters, the absolute values of the estimated parameters are in general not comparable across literature. Hence, we focus on the signs of the estimated parameters which are comparable in most cases: a positive parameter indicates risk/ambiguity aversion, a negative parameter indicates risk/ambiguity loving, and zero indicates risk/ambiguity neutrality.

Among the previous literature, one typical work is Krahnen et al. (2014). In one of the specifications, the authors specify both utility functions with power functional forms (CRRA+CRAA), comparable to our Specification 1. The authors find that the estimated risk parameter has a zero median, indicating risk neutrality overall. The estimated ambiguity parameter has a positive median, indicating ambiguity aversion overall. These findings are similar to ours. However, our estimations seem to be more accurate and reliable, since Krahnen et al. (2014) do not explicitly specify the beliefs, which may lead to (probably downwards) biased results under the incorrect belief characterizations as discussed above.

**Specification 2: CRRA and CAAA**. In this specification, the utility function $\phi$ takes the following form:

$$\phi(y) = \begin{cases} \dfrac{-e^{-\delta y}}{\delta}; & \text{if } \delta \neq 0 \quad (33) \\ \\ y; & \text{if } \delta = 0 \quad (34) \end{cases}$$

where $\phi$ exhibits constant absolute ambiguity attitude (CAAA): $-\frac{\phi''(y)}{\phi'(y)} = \delta$, where $\delta > 0$ indicates ambiguity aversion, $\delta = 0$ indicates ambiguity neutral, and $\delta < 0$ indicates ambiguity seeking. In addition, $\phi$ is increasing in $y$ for any value of $y$, and continuously twice-differentiable with respect to $y$.

Analogously, re-writing the expression of the subjective CE of an ambiguity choice list (Equation 25) with the specified utilities in Equation (27) and (33)/(34) yields:

$$X_{i,A_n} = \begin{cases} \left\{ \dfrac{-log\left[\sum_{\theta=0}^{1} \text{pmf}(\hat{\alpha}_{i,n}, \hat{\beta}_{i,n}, \theta) \cdot exp(-\delta_i(1+\theta))\right]}{\delta_i} - 1 \right\}^{\frac{1}{1-\rho_i}}; & \text{if } \delta_i \neq 0 \quad (35) \\ \left[ \displaystyle\sum_{\theta=0}^{1} \text{pmf}(\hat{\alpha}_{i,n}, \hat{\beta}_{i,n}, \theta) \cdot (1+\theta) - 1 \right]^{\frac{1}{1-\rho_i}}; & \text{if } \delta_i = 0 \quad (36) \end{cases}$$

The corresponding degenerated form for the risk choice lists is identical to Equation (32), which is a special case of Equation (35) and (36). For each subject $i$, we estimate her ambiguity attitude ($\delta_i$) and her risk attitude ($\rho_i$) simultaneously, applying nonlinear regression based on Equation (35) or (36) (after adding an error term to the equation), using all her responses in the choice lists. Finally, the estimated parameters, $\hat{\rho}_i$ and $\hat{\delta}_i$, are obtained from one of the two estimations (one based on Equation 30, and one based on Equation 31) which achieves the better goodness of fit (i.e. the higher $R^2$). In fact, only two subjects turn out to be better estimated by $\hat{\delta}_i = 0$ in terms of goodness of fit, for whom the estimation results are thus obtained from Equation (36). Two subjects have no estimation results at all, since no variation is found in the data of each. For the remaining 98 subjects, their estimated parameters are obtained from the regression based on Equation (35). Overall, the estimation tends to fit the data very well: the $R^2$ is on average 0.94.

Table 4 Specification 2a report the results of $\hat{\rho}_i$ and $\hat{\delta}_i$ based on Specification 2. In the regressions, the estimated personal beliefs are used as input. The results show that on average $\hat{\rho}_i$ is slightly negative but insignificantly different from zero. Subjects with positive $\hat{\rho}_i$ values are nearly as many as subjects with negative ones. Overall, $\hat{\rho}_i$ tends to be zero, indicating risk neutrality. This is robust to the findings under Specification 1a. This implies that the transition of $\phi$ function from a power utility (CRAA) to an exponential utility (CAAA) hardly affects the estimation of the risk attitude parameter. This is understandable, since the specification of $u$, which governs the risk attitude, is identical in Specification 1a and 2a. The transition of $\phi$ affects the estimation of the ambiguity attitude parameter more evidently. The mean value of $\hat{\delta}_i$ is equal to 10.552, slightly significant. In comparison with the significantly positive mean value in Specification 1a, $\hat{\delta}_i$s in 2a display overall milder ambiguity aversion. However, heterogeneity still sustains. The proportion of the ambiguity averse subjects ($\hat{\delta}_i > 0$, N=67) and the proportion of the ambiguity loving subjects ($\hat{\delta}_i < 0$, N=31) are quite robust to Specification 1a. This implies that, in comparison with the estimated $\hat{\delta}_i$ under the power utility function (CRAA), the results under the exponential utility function (CAAA) seems to be more concentrated around zero. However, the sign of each $\hat{\delta}_i$, which reflects whether a subject is ambiguity averse or ambiguity loving, is consistent. In other words, when considering which specification is to adopted for analyses, whether choosing CRAA or CAAA probably does not affect the investigation for whether a subject is ambiguity averse or ambiguity loving, but may affect the magnitude of the ambiguity aversion or ambiguity loving.

Analogously, to see more clearly the importance of using the "correct" personal belief characterizations in attitude estimation, we re-run the regressions using the constructed all-neutral belief dynamics. The results are reported in Table 4 Specification 2b. In rows related to risk attitude, $\hat{\rho}_i$ has a mean value equal to $-0.388$ in 2b, different from the mean value $-0.520$ in 2a. Yet, such difference is small and insignificant (see"2a minus 2b" column). As for ambiguity attitude, the $\hat{\delta}_i$ tends to be slightly underestimated in Specification 2b compared with in 2a. In 2b, the mean value of $\hat{\delta}_i$ is positive but insignificant. This result is not too much different from that in 2a, where $\hat{\delta}_i$ is positive and only significant at 10%. Figure 5b illustrates the transition of $\hat{\delta}_i$s from 2b to 2a. It seems that most $\hat{\delta}_i$s take similar values under the two different belief characterizations. The magnitude of the overestimations (left half of the graph) and the magnitude of the underestimations (right half of the graph) tend to be smaller, in comparison with the transition from 1b to 1a (in Figure 5a). This implies that the accuracy of the ambiguity attitude estimation under CAAA (Specification 2) is less sensitive to the correctness of the belief characterizations in comparison with the estimation under CRAA (Specification 1).

Some previous literature which applies the KMM model assumes similar utility specifications as we do in Specification 2. A closely related work is Cubitt et al. (2018). The authors specify the risk utility function $u$ with a power form (CRRA) and the ambiguity utility function $\phi$ with an exponential form (CAAA). Moreover, the authors also parameterize and estimate personal beliefs, which makes their paper even more comparable to our Specification 2. The authors find that overall the subjects are risk averse: the median of the risk attitude coefficient is positive, and obviously more subjects can be categorized as risk averse than as risk loving based on the risk attitude coefficient. As for the ambiguity attitude coefficient, the authors also find ambiguity aversion: the median of the ambiguity attitude coefficient is significantly positive. We reach the similar results in terms of ambiguity attitude, but different results in term of risk attitude. This may be explained by the different specific forms of the utility functions that two papers adopt. Specifically, Cubitt et al. (2018) adopt a risk utility function which tends to violate the requirement of the KMM model (see Theorem 1 in Klibanoff et al. (2005) and Izhakian and Benninga (2011)). Our paper fixes this problem. Moreover, Cubitt et al. (2018) estimate beliefs simultaneously with attitude parameters, and thus the belief estimation may be confounded by the attitude estimation. In contrast, we estimate beliefs independently from attitudes, which tends to provide more accurate results. At last, in Cubitt et al. (2018) the second-order probability has a binary support, while in our model there are 101 supports. The generality of our setting is also an improvement compared with Cubitt et al. (2018).

With the results from both Specification 1 and 2, we can reach the following conclusions: the attitude parameter estimations using the recovered personal beliefs tend to be robust under Specification 1 (CRRA+CRAA) and Specification 2 (CRRA+CAAA): Both specifications fit the data very well (high $R^2$ in both specifications). On average, the parameter governing a subject's risk attitude ($\rho_i$) tends to be zero, indicating risk neutrality; the parameter governing a subject's ambiguity attitude ($\delta_i$) tends to be positive, indicating ambiguity aversion. Heterogeneity prevails in both parameters and in both specifications. If beliefs are mistakenly characterized, in general $\delta_i$ is likely to be underestimated (downwards biased) under both Specification 1 and 2, especially under Specification 1. For individual subjects, the biases are heterogeneous: both underestimated and overestimated $\delta_i$s are prevalent across subjects under both specifications. These results consolidate the necessity of the correct recovery of the personal beliefs, prior to the analysis of attitude estimation. With mistakenly characterized beliefs, the estimations of attitude parameters are very likely to be biased for individual subjects, no matter which mainstream utility functions are used.

## 4.3 Estimation of price deviations

In the previous chapters, we estimate the parameters governing a subject's belief formation and updating ($\hat{\alpha_{i,0}}$, $\hat{\beta}_{i,0}$, $\hat{\gamma}_i^w$ and $\hat{\gamma}_i^b$). Using the recovered belief dynamics as inputs, we further estimate the risk attitude ($\hat{\rho}_i$) and the ambiguity attitude ($\hat{\delta}_i$) under two different specifications of utility functions. In this chapter, we use the belief estimation results and the attitude estimation results under Specification 1 (1a in Table 4) to derive the estimated total price deviation and its three components (i.e. belief effect, risk premium, and ambiguity premium) for each subject when she prices the ambiguous asset as defined in this paper. The estimations at two specific points in time are our interest: before any draw has occurred ($n = 0$), and after 15 draws have occurred ($n = 15$). The nice feature of the KMM model that beliefs and attitudes are separable by construction facilitates a clean measurement of the three components of the total price deviation. This also enables a comparison between the estimated price deviations based on the parametric analysis (in this chapter) and the price deviations based on the non-parametric analysis (in Chapter 3).

Analogously, we first define an all-neutral benchmark-CE, which is induced by neutral beliefs, neutral risk attitude, and neutral ambiguity attitude. The neutral benchmark beliefs are characterized by a uniformly distributed initial prior and the perfect employment of Bayes' rule in belief updating. This means that the benchmark belief is parameterized by $\alpha_{i,0} = \beta_{i,0} =$

$\gamma_i^w = \gamma_i^b = 1$, where subscript $i$ indicates that this benchmark belief dynamic is generated based on the draw history observed by subject $i$. The benchmark-CE is equal to the mean of the benchmark belief. This is due to the fact that the first-order probability follows a Bernoulli distribution, hence the mean of the second-order probability (the mean of the benchmark belief distribution) is equal to the expected value of the ambiguous asset (the benchmark CE), provided that risk attitude and ambiguity attitude are both neutral. Thus, we can write:

$$CE_{i,n}^{neutral} = \frac{k_{i,n} + 1}{n + 2} \tag{37}$$

**Estimated total price deviation**. Analogous to Equation (3), we define the total price deviation as the difference between the benchmark CE and the estimated conceived CE of a subject.

$$TotalDev_{i,n}^{est} = \frac{CE_{i,n}^{neutral} - CE_{i,n}^{est}}{CE_{i,n}^{neutral}} \tag{38}$$

$$\text{where } CE_{i,n}^{est} = \left\{ \left[ \sum_{\theta=0}^{1} \text{pmf}(\hat{\alpha}_{i,n}, \hat{\beta}_{i,n}, \theta) \cdot (1+\theta)^{1-\hat{\delta}_i} \right]^{\frac{1}{1-\hat{\delta}_i}} - 1 \right\}^{\frac{1}{1-\hat{\rho}_i}} \tag{39}$$

$$\theta = 0, 0.01, \cdots 1; \quad i = 1, 2 \cdots N; \quad n = 0, 15$$

Equation (39) estimates the conceived CE of subject $i$ following Equation (30). The estimated total price deviation is also defined as a relative term, relative to the benchmark CE. The results of $TotalDev_{i,n}^{est}$ are reported in Table 5 Panel C. The results are robust in comparison with $TotalDev_{i,n}$ in Table 3 Panel A: Overall, the estimated total price deviation is strongly positive before any draw has occurred, and turn to be negative after 15 draws are implemented. This indicates the average price increases as information accumulates and ambiguity resolves. On an individual level, Figure 6 plots the distribution of $TotalDev_{i,n}^{est}$ across subjects, by the number of draws ($n = 0$ and $n = 15$). Similar to $TotalDev_{i,n}$ in Figure 3, there exists heterogeneity. In addition, the distribution also shifts down slightly as the number of draws increases to 15.

Since a subject's total price deviation can be decomposed into three components, we can write:

$$TotalDev_{i,n}^{est} = BE_{i,n}^{est} + RP_{i,n}^{est} + AP_{i,n}^{est} \tag{40}$$

This is analogous to Equation (4). $BE_{i,n}^{est}$, $RP_{i,n}^{est}$, and $AP_{i,n}^{est}$ denote the estimated belief effect, estimated risk premium, and the estimated ambiguity premium of subject $i$ after observing $n$ draws, respectively. To distinguish the variables in this chapter from those in Chapter 3, the superscripts "est" (meaning "estimated") are added. To proceed, we compute the three components one after another.

**Estimated belief effect**. Analogous to Equation (5), the estimated belief effects can be derived as follow:

$$BE_{i,n}^{est} = \left[ CE_{i,n}^{neutral} - \frac{\hat{\alpha}_{i,n}}{\hat{\alpha}_{i,n} + \hat{\beta}_{i,n}} \right] \Big/ CE_{i,n}^{neutral} \tag{41}$$

$$i = 1, 2 \cdots N; \quad n = 0, 15$$

where $\hat{\alpha}_{i,n}$ and $\hat{\beta}_{i,n}$, governing the personal belief, can be recovered from Equation (11) and (12) using the estimated belief parameters and draw history, respectively. The term $\hat{\alpha}_{i,n}/(\hat{\alpha}_{i,n}+\hat{\beta}_{i,n})$ represents the mean value of the estimated personal belief. This is also equal to the conceived CE of subject $i$ given her estimated belief, assuming risk neutrality and ambiguity neutrality. Therefore, $BE^{est}_{i,n}$ is purely belief relevant, independent from attitudes. This can also be justified by the fact that the attitude parameters, $\hat{\rho}_i$ and $\hat{\delta}_i$, do not enter the belief effect formula. Analogous to $BE_{i,n}$, $BE^{est}_{i,n}$ also provides a clean measurement of the effects ascribed to the personal beliefs, without confounding from attitudes. The results are reported in Table 5 Panel C. On average, the estimated belief effect tends to be close to zero at $n = 0$ (mean=-0.041, median=0.000), and turns to be negative at $n = 15$ (mean=-0.206, median=-0.065). This is similar to the tendency of $BE_{i,n}$ in Table 3 Panel A. In addition, the negative estimated belief effects after 15 draws may be explained by subjects' stickiness to their current beliefs, rather than their optimism in beliefs. This is supported by the fact that 79 out of 102 subjects are estimated by $\hat{\gamma}^w_i < 1$ and $\hat{\gamma}^b_i < 1$, which indicates stickiness to current beliefs in belief updating. Figure 6 illustrates the distributions of $BE^{est}_{i,n}$ at $n = 0$ and $n = 15$, respectively. Again, heterogeneity prevails. However, $BE^{est}_{i,0}$ seems to be more compressed than $BE_{i,0}$ in Figure 3. This may be partly explained by the application of the estimated beliefs: the belief estimation model "smooth" the belief dynamic of each subject. The fitted values derived from the regression model (i.e. the estimated beliefs) are thus less heterogeneous.

**Estimated risk premium**. Analogous to Equation (6), the estimated risk premium can be derived as follow:

$$RP^{est}_{i,n} = \left[ \frac{\hat{\alpha}_{i,n}}{\hat{\alpha}_{i,n}+\hat{\beta}_{i,n}} - \left( \frac{\hat{\alpha}_{i,n}}{\hat{\alpha}_{i,n}+\hat{\beta}_{i,n}} \right)^{\frac{1}{1-\hat{\rho}_i}} \right] \Big/ CE^{neutral}_{i,n} \tag{42}$$

$$i = 1, 2 \cdots N; \quad n = 0, 15$$

The term $\hat{\alpha}_{i,n}/(\hat{\alpha}_{i,n}+\hat{\beta}_{i,n})$ follows the explaination above. The term $[\hat{\alpha}_{i,n}/(\hat{\alpha}_{i,n}+\hat{\beta}_{i,n})]^{1/(1-\hat{\rho}_i)}$ can be interpreted as subject $i$'s conceived CE under her personal belief and modified by her estimated risk attitude ($\hat{\rho}_i$), assuming ambiguity neutral ($\hat{\delta}_i = 0$). Therefore, the difference between the two terms is solely ascribed to her risk attitude $\hat{\rho}_i$. It is easy to check that $\hat{\rho}_i = 0$ leads to $RP^{est}_{i,n} = 0$. Both terms inside the big bracket are computed based on the personal beliefs. Thus, the subtraction partials out the effect of the personal beliefs, leaving $RP^{est}_{i,n}$ a clean measurement of effects ascribed to a subject's risk attitude. The results of $RP^{est}_{i,n}$ are reported in Table 5 Panel C. Overall, the estimated risk premium tends to be close to zero at $n = 0$ and $n = 15$. This is consistent with findings of $RP_{i,n}$ in Table 3 Panel A. The distributions of $RP^{est}_{i,n}$ are illustrated in Figure 6, for $n = 0$ and $n = 15$, respectively. The heterogeneity is prevalent at both points in time, similar to the results from Figure 3.

**Estimated ambiguity premium**. At last, analogous to Equation (7), the estimated ambiguity premium can be derived as follow:

$$AP_{i,n}^{est} = \left[ \left( \frac{\hat{\alpha}_{i,n}}{\hat{\alpha}_{i,n} + \hat{\beta}_{i,n}} \right)^{\frac{1}{1-\hat{\rho}_i}} - CE_{i,n}^{est} \right] \Big/ CE_{i,n}^{neutral} \tag{43}$$
$$i = 1, 2 \cdots N; \quad n = 0, 15$$

where $CE_{i,n}^{est}$, derived in Equation (39), denotes subject $i$'s conceived CE under her personal belief, modified by her risk attitude ($\hat{\rho}_i$) and her ambiguity attitude ($\hat{\delta}_i$). As explained above, the first term on the RHS can be interpreted as $i$'s conceived CE under her personal belief, modified by her risk attitude ($\hat{\rho}_i$) but assuming ambiguity neutrality ($\hat{\delta}_i = 0$). Therefore, $AP_{i,n}^{est}$ is solely ascribed to her (non-neutral) ambiguity attitude $\hat{\delta}_i$. It is easy to show that $\hat{\delta}_i = 0$ leads to $AP_{i,n}^{est} = 0$. In addition, both terms inside the big bracket are computed based on the personal beliefs. Thus, the subtraction partials out the effect of the personal beliefs, leaving $AP_{i,n}^{est}$ a clean measurement of effects ascribed to a subject's ambiguity attitude.

The results of $AP_{i,n}^{est}$ are reported in Table 5 Panel C. Before any draw has occurred, the estimated ambiguity premium is on average strongly positive (mean=0.148, median=0,085). After 15 draws, the average ambiguity premium decreases (mean=0.118, median=0.052). This is consistent with the intuition: as ambiguity gradually resolves, subjects tend to require less compensation for the exposure to ambiguity, which drags down the ambiguity premiums. Such trend is robust to the findings of $AP_{i,n}$ in Table 3 Panel A. However, after 15 draws, $AP_{i,15}^{est}$ is still strongly positive, while $AP_{i,15}$ tends to be zero. This may be explained by characterizations of the estimated beliefs after 15 draws. As Figure 4b implies, most subjects hold the beliefs which still reflect some degree of ambiguity, i.e. the belief curves are still rather flat after 15 draws, and ambiguity is still conceived to be strong. Therefore, we observe positive ambiguity premium based on this belief estimation at 15 draws. It seems that conceived ambiguity degree decreases more quickly in the observed beliefs (directly from the experiment data) than in the estimated beliefs (the fitted values). Therefore, $AP_{i,n}$ decreases more rapidly than $AP_{i,n}^{est}$ as the number of draws increases to 15.

In addition, Figure 6 illustrates the distributions of $AP_{i,n}^{est}$ at $n = 0$ and $n = 15$, respectively. Heterogeneity prevails at both points in time, but decreases over time. This is robust to the findings of $AP_{i,n}$ in Figure 3.

As for the proportion of each components in the total price deviation, before any draw has occurred, ambiguity premium tend to play a dominating role among the three. After 15 draws have been implemented, the belief effect tend to override the dominating position of the ambiguity premium. Risk premium is small at both points in time. In general, the price deviations derived from the estimated belief and attitude parameters tend to be robust to the price deviations directly derived from the experiment data.

# 5 Conclusion

This paper uses laboratory experiments to study how the presence of ambiguity affects subjects' pricing strategies. In particular, we investigate for each individual subject how her valuation of a lottery with unknown winning probability deviates from an all-neutral benchmark valuation. As in previous empirical literature which studies price deviations under ambiguity, we also observe a price discount in the presence of ambiguity. Our paper builds on these results and moves forward: unlike previous studies, our experiment design allows us to cleanly disentangle the total price deviation into three separable components: belief effect, risk premium, and ambiguity premium. This separation allows us to further investigate to what extent the price deviation is ascribed to beliefs, and to what extent it is ascribed to ambiguity attitude.

The results show that in case of a high degree of ambiguity, on average the ambiguity premium accounts for the biggest proportion of the price deviation. Overall, subjects display substantial aversion against ambiguity. On the other hand, the belief effect plays a smaller role. There exists neither substantial pessimism nor substantial optimism in beliefs. As information accumulates, i.e. the degree of ambiguity decreases, the ambiguity premium decreases, and belief effects emerge. The emerging belief effects are mainly driven by the under-adjustment in belief updating (in comparison with Bayes' rule), and not by pessimism or optimism in subjects' beliefs. In other words, in a dynamic setting, beliefs tend to affect pricing through subjects' sticky belief updating rules, and not through subjects' biased views regarding possible scenarios per se. Therefore, both factors, belief and attitude towards ambiguity, are important factors in pricing under ambiguity.

In addition, for each subject, we estimate the parameter governing risk aversion ($\rho$) and the parameter governing ambiguity aversion ($\delta$) under the KMM model. The KMM model realizes the parametric separation of belief, risk attitude, and ambiguity attitude. Together with our experiment design, we are able to cleanly estimate the two attitude parameters, with belief effects completely partialled out. The parameter estimations show that on average subjects display slight risk loving and strong ambiguity aversion, i.e. the mean of $\hat{\rho}$ is slightly negative, and the mean of $\hat{\delta}$ is significantly positive ($\hat{\rho} = -0.5$ and $\hat{\delta} = 52.3$ in a CRRA+CRAA specification; $\hat{\rho} = -0.5$ and $\hat{\delta} = 10.6$ in a CRRA+CAAA specification).

The investigation of belief effects also implies that in case that beliefs are not correctly characterized, the results of ambiguity aversion are probably biased. This is shown in the parameter estimation: the estimated ambiguity aversion ($\delta$) under the neutral benchmark beliefs is different from the estimated ambiguity aversion under the recovered beliefs, especially on the level of individual subjects. This confirms the necessity of separating beliefs and attitudes in settings involving ambiguity.

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Table 1: Experiment procedure

This table reports the main procedure of a complete experiment session. Some games may show up only in some sessions, instead of all sessions, but the time-line applies to all sessions. $n$ denotes the number of draws already implemented. "$R$" denotes risk choice list, in which Option B is some purely risky lottery. "$A$" denotes ambiguity choice lists, in which Option B is the ambiguous lottery (operationalized by the ambiguous urn). In risk/ambiguity choice lists, a subject reports her conceived reservation value (certainty equivalent) of the lottery in Option B. "$G$" denotes guess game, in which subjects guess the number of white balls in the ambiguous urn.

| **n=0** → | **n=1** → | ⋯ | **n=14** → | **n=15** |
|---|---|---|---|---|
| $R50$, $R25$, $R75$ | | | | |
| ↓ | | | | |
| $G_0$ | $G_1$ | ⋯ | $G_{14}$ | $G_{15}$ |
| ↓ | ↓ | | ↓ | ↓ |
| $A_0$ | $A_1$ | ⋯ | $A_{14}$ | $A_{15}$ |
| ↓ | ↓ | | ↓ | ↓ |
| Asset Trading | Asset Trading | ⋯ | Asset Trading | $R_0$, $R_{14}$, $R_{15}$, $R_{ml}$, $R_{bay}$ |
| ↓ | ↓ | | ↓ | ↓ |
| $1^{st}$ draw | $2^{nd}$ draw | ⋯ | $15^{th}$ draw | Selected choice lists are played for real; Questionnaire; Final payment. |

**Notations**

| | Type of lottery/urn | No. of white balls (W) | No. of black balls | Total | Description |
|---|---|---|---|---|---|
| *Choice list: lottery in Option B* | | | | | |
| $R50$ | risky | 50 | 50 | 100 | |
| $R25$ | risky | 25 | 75 | 100 | |
| $R75$ | risky | 75 | 25 | 100 | |
| $R_0$ | risky | subject's response in $G_0$ | 100-W | 100 | |
| $R_{14}$ | risky | subject's response in $G_{14}$ | 100-W | 100 | |
| $R_{15}$ | risky | subject's response in $G_{15}$ | 100-W | 100 | |
| $R_{ml}$ | risky | ML update* | 100-W | 100 | |
| $R_{bay}$ | risky | Bayesian update$ | 100-W | 100 | |
| $A_n$ | ambiguous | unknown | unknown | 100 | played after $n$ draw(s) |
| *Guess game: the urn* | | | | | |
| $G_n$ | ambiguous | unknown | unknown | 100 | played after $n$ draw(s) |

*Suppose that a subject observes in total $T$ draws, with $k$ units of them being white draws, her ML update (maximum likelihood update) is equal to $100k/T$.

$Assume that a subject starts with a uniformly distributed initial belief, and updates it in response to the draw information by employing Bayes' rule. The Bayesian update here refers to the mean value of a subject's updated belief distribution after she observes all draws.

Table 2: No. of subjects per Session/Market

This table summarizes the information of subjects participating in each experiment session, reported in Column (1)-(4). Column (5)-(7) report the included guess games, risk choice lists, and ambiguity choice lists in each session, respectively. Column (6)/(7) also reports the number of risk/ambiguity choice lists played for real (i.e. the earnings from these choice lists enter a subject's final payment) in brackets "[ ]". Column (8) reports the draw history path generated in each session. Column (9) reports whether the asset trading is included in each session.

| Session ID (1) | Market ID (2) | Subject ID (3) | No. of subjects (4) | Guess games (5) | Risk choice lists [No. of played for real] (6) | Ambiguity choice lists [No. of played for real] (7) | Draw history* (8) | Asset trading (9) |
|---|---|---|---|---|---|---|---|---|
| I | 1 | 1-13 | 13 | $G_0, G_1 \cdots G_{14}$ | $R50, R_0, R_{14}, R_{ml}$ [4] | $A_0$ [1] | Path 1 | Yes |
| II | 2 | 14-20 | 7 | $G_0, G_1 \cdots G_{15}$ | $R50, R_0, R_{15}, R_{ml}, R_{bay}$ [5] | $A_0, A_{15}$ [2] | Path 2 | Yes |
| | 3 | 21-27 | 7 | | | | Path 3 | |
| III | 4 | 28-34 | 7 | $G_0, G_1 \cdots G_{15}$ | $R50, R_0, R_{15}, R_{ml}, R_{bay}$ [5] | $A_0, A_{15}$ [2] | Path 4 | Yes |
| | 5 | 35-41 | 7 | | | | Path 5 | |
| IV | 6 | 42-48 | 7 | $G_0, G_1 \cdots G_{15}$ | $R50, R25, R75, R_0, R_{15}$ [5] | $A_0, A_2 \cdots A_{15}$ [5] | Path 6 | Yes |
| | 7 | 49-55 | 7 | | | | Path 7 | |
| V | 8 | 56-62 | 7 | $G_0, G_1 \cdots G_{15}$ | $R50, R25, R75, R_0, R_{15}$ [5] | $A_0, A_2 \cdots A_{15}$ [5] | Path 8 | Yes |
| | 9 | 63-69 | 7 | | | | Path 9 | |
| VI | 10 | 70-76 | 7 | $G_0, G_1 \cdots G_{15}$ | $R50, R25, R75, R_0, R_{15}$ [5] | $A_0, A_{15}$ [2] | Path 10 | Yes |
| | 11 | 77-83 | 7 | | | | Path 11 | |
| VII | - | 84-102 | 19 | $G_0, G_1 \cdots G_{15}$ | $R50, R25, R75, R_0, R_{15}$ [3] | $A_0, A_2 \cdots A_{15}$ [6] | Path 12-30 | No |
| | | | *Total*=102 | | | | *Total*=30 draw history paths | |

For markets 1-11, each market observes one independent draw history, i.e. subjects in the same market observe the same draws. In Session VII, each subject observes one independent draw history.

Table 3: Belief effect, risk premium and ambiguity premium

Panel A summarizes the results of the total price deviation, the belief effect, the risk premium, and the ambiguity premium when a subject prices the ambiguous asset, respectively. For subject $i$, the total price deviation ($TotalDev_{i,n}$) computes the price deviation in comparison with an all-neutral benchmark, when she prices the ambiguous asset in the ambiguity choice list $A_n$ ($n = 0, 15$). It is computed based on Equation (3). The total price deviation is then decomposed into three components: the belief effect (deviation ascribed to personal beliefs, denoted as $BeliefDev_{i,n}$), the risk premium (deviation arising from non-neutral risk attitude, denoted as $RiskDev_{i,n}$), and the ambiguity premium (deviation arising from non-neutral ambiguity attitude, denoted as $AmbDev_{i,n}$). They are computed based on Equation (5)-(7), respectively. The statistical summaries of the four variables are reported in Panel A, by the number of draws. Panel B reports the number of subjects by the range of each variable reported in Panel A. Panel C reports the results of $\hat{\psi}_i^p$ and $\hat{\psi}_i^s$. Belief effects can be induced by a subject's pessimism/optimism and/or the stickiness to her current belief (lack of responding to new draw information). These two inducements of the belief effects, parameterized by $\psi_i^p$ and $\psi_i^s$ respectively, are estimated based on Equation (9). $\psi_i^p = 1$ indicates neither pessimism nor optimism is found in belief updating; $\psi_i^p > 1 (< 1)$ indicates pessimism (optimism). $\psi_i^s = 1$ indicates no stickiness to current beliefs (fully responding to new information as ML updates); $\psi_i^s < 1$ indicates stickiness to current beliefs (lack of updating); $\psi_i^s > 1$ indicates overly responding to new information.

**Panel A: Total price deviation and its three components**

| | n=0 (before any draw has occurred) | | | | n=15 (after 15 draws) | | | |
|---|---|---|---|---|---|---|---|---|
| | mean | sd. | median | N | mean | sd. | median | N |
| $TotalDev_{i,n}$ | 0.134*** | 0.389 | 0.040 | 102 | -0.152** | 0.566 | -0.027 | 89 |
| $BE_{i,n}$ | 0.007 | 0.222 | 0.000 | 102 | -0.083* | 0.454 | 0.000 | 89 |
| $RP_{i,n}$ | 0.020 | 0.313 | 0.000 | 102 | -0.029 | 0.447 | 0.000 | 89 |
| $AP_{i,n}$ | 0.107*** | 0.323 | 0.003 | 102 | -0.040 | 0.429 | 0.000 | 89 |

**Panel B: No. of subjects by the range of price deviation**

| | n=0 (before any draw has occurred) | | | | n=15 (after 15 draws) | | | |
|---|---|---|---|---|---|---|---|---|
| | $TotalDev_{i,0}$ | $BE_{i,0}$ | $RP_{i,0}$ | $AP_{i,0}$ | $TotalDev_{i,15}$ | $BE_{i,15}$ | $RP_{i,15}$ | $AP_{i,15}$ |
| variable$< 0$ | 32 | 26 | 42 | 26 | 51 | 32 | 41 | 40 |
| variable$= 0$ | 13 | 42 | 18 | 25 | 8 | 28 | 9 | 25 |
| variable$> 0$ | 57 | 34 | 42 | 51 | 30 | 29 | 39 | 24 |
| Total ($N$) | 102 | 102 | 102 | 102 | 89 | 89 | 89 | 89 |

**Panel C: Belief effect after 15 draws ($BE_{i,15}$): optimism or stickiness**

| | mean | s.d | N | No. of subjects, for whom $H_0 : \psi_i^p = 1$ can be rejected at 5% |
|---|---|---|---|---|
| $\hat{\psi}_i^p$: optimism | 0.984 | 0.165 | 89 | 8 |
| | mean | s.d | N | No. of subjects, for whom $H_0 : \psi_i^s = 1$ can be rejected at 5% |
| $\hat{\psi}_i^s$: stickiness to current beliefs | 0.357*** | 0.261 | 89 | 79 |

In Panel A and C, the significance tests in the "mean" columns refer to the two-sided t-tests with the null hypothesis that the mean value is equal to zero. $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Table 4: Estimation of attitude parameters: under different utilities and beliefs

This table reports the estimated risk attitude parameter ($\hat{\rho}_i$) and the ambiguity attitude parameter ($\hat{\delta}_i$) under different utility specifications and different belief characterizations. In Specification 1 (CRRA+CRAA), The utility function governing the risk attitude/ambiguity attitude is specified in Equation (27)/(30)-(31). In Specification 2 (CRRA+CAAA), the utility function governing the risk attitude/ambiguity attitude is specified in Equation (27)/(35)-(36). In each Specification, for each subject, the estimation is conducted under two different beliefs, respectively: either using the estimated personal beliefs (indexed by "a") or assuming the all-neutral beliefs (indexed by "b"). The differences of the estimated values of a parameter, induced by assuming/using different beliefs, is summarized in column "1a minus 1b" for Specification 1 and in column "2a minus 2b" for Specification 2. All results related to $\hat{\rho}_i$ ($\hat{\delta}_i$) are reported in the upper (lower) part of the table.

| | Specification 1a<br>CRRA: $u(c) = 1 + c^{1-\rho}$<br>CRAA: $\phi(y) = \frac{y^{1-\delta}-1}{1-\delta}$<br>Estimated beliefs | | | Specification 1b<br>CRRA: $u(c) = 1 + c^{1-\rho}$<br>CRAA: $\phi(y) = \frac{y^{1-\delta}-1}{1-\delta}$<br>All-neutral beliefs | | | 1a minus 1b[§] | | Specification 2a<br>CRRA: $u(c) = 1 + c^{1-\rho}$<br>CAAA: $\phi(y) = \frac{-e^{-\delta y}}{\delta}$<br>Estimated beliefs | | | Specification 2b<br>CRRA: $u(c) = 1 + c^{1-\rho}$<br>CAAA: $\phi(y) = \frac{-e^{-\delta y}}{\delta}$<br>All-neutral beliefs | | | 2a minus 2b[§] | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\hat{\rho}_i$ | mean | s.d | N | mean | s.d | N | mean | N | mean | s.d | N | mean | s.d | N | mean | N |
| All | -0.522 | 3.258 | 100 | -0.365** | 1.449 | 100 | -0.157 | 100 | -0.520 | 3.258 | 100 | -0.388*** | 1.453 | 100 | -0.132 | 100 |
| $> 0$ | 0.176 | 0.141 | 49 | 0.145 | 0.123 | 52 | 0.261 | 53 | 0.173 | 0.141 | 50 | 0.146 | 0.124 | 50 | 0.253 | 56 |
| $= 0$ | 0.000 | 0.000 | 1 | 0.000 | 0.000 | 0 | 0.000 | 8 | 0.000 | 0.000 | 1 | 0.000 | 0.000 | 0 | 0.000 | 6 |
| $< 0$ | -1.217 | 4.521 | 50 | -0.918 | 1.950 | 48 | -0.758 | 39 | -1.237 | 4.565 | 49 | -0.922 | 1.915 | 50 | -0.719 | 38 |
| $\hat{\delta}_i$ | mean | s.d | N | mean | s.d | N | mean | N | mean | s.d | N | mean | s.d | N | mean | N |
| All | 52.255** | 255.591 | 100 | 13.748 | 205.439 | 100 | 38.507* | 100 | 10.552* | 54.603 | 100 | 9.105 | 67.718 | 100 | 1.446 | 100 |
| $> 0$ | 81.792 | 308.226 | 67 | 38.312 | 214.973 | 67 | 70.854 | 62 | 19.574 | 62.656 | 67 | 20.396 | 68.213 | 67 | 12.890 | 55 |
| $= 0$ | 0.000 | 0.000 | 4 | 0.000 | 0.000 | 0 | 0.000 | 2 | 0.000 | 0.000 | 2 | 0.000 | 0.000 | 0 | 0.000 | 2 |
| $< 0$ | -8.778 | 26.554 | 29 | -36.125 | 177.280 | 33 | -15.061 | 36 | -8.267 | 25.435 | 31 | -13.819 | 61.493 | 33 | -13.124 | 43 |

CRRA stands for constant relative risk aversion; CRAA stands for constant relative ambiguity aversion; CAAA stands for constant absolute ambiguity aversion. [§]1a minus 1b refers to the parameter estimated based on specification 1a minus the same parameter estimated based on Specification 1b. Analogous for "2a minus 2b". The significance tests are only conducted for the mean values of the full sample (the "All" rows): two-sided t-tests with a null hypothesis that the mean value is equal to zero. *** $p < 0.01$; ** $p < 0.05$ *$p < 0.1$.

Table 5: Estimated total price deviation and its three components: under Specification 1

The table reports the estimation results of the total price deviation and its three components. All four variables are estimations based on the estimated beliefs and estimated attitude parameters under Specification 1 of utility functions. For subject $i$, the estimated total price deviation ($TotalDev_{i,n}^{est}$) computes the price deviation in comparison with an all-neutral benchmark, when subject $i$ prices the ambiguous asset in the ambiguity choice list $A_n$ ($n = 0, 15$). $TotalDev_{i,n}^{est}$ is computed based on Equation (38). The estimated total price deviation is then decomposed into three components: the belief effect (deviation ascribed to personal beliefs, denoted as $BE_{i,n}^{est}$), the risk premium (deviation arising from non-neutral risk attitude , denoted as $RP_{i,n}^{est}$), and the ambiguity premium (deviation arising from non-neutral ambiguity attitude, $AP_{i,n}^{est}$). They are computed based on Equation (41), (42) and (43), respectively.

| | n=0 (before any draw has occurred) | | | | n=15 (after 15 draws) | | | |
|---|---|---|---|---|---|---|---|---|
| | mean | sd. | median | N | mean | sd. | median | N |
| $TotalDev_{i,n}^{est}$ | 0.087** | 0.353 | 0.035 | 100 | -0.116** | 0.525 | -0.014 | 88 |
| $BE_{i,n}^{est}$ | -0.041* | 0.208 | 0.000 | 102 | -0.206*** | 0.445 | -0.065 | 89 |
| $RP_{i,n}^{est}$ | -0.019 | 0.242 | 0.000 | 100 | -0.026 | 0.342 | 0.000 | 88 |
| $AP_{i,n}^{est}$ | 0.148*** | 0.246 | 0.085 | 100 | 0.118*** | 0.206 | 0.052 | 88 |

The significance tests refer to two-sided t-tests with a null hypothesis that the mean value is equal to zero. *** $p < 0.01$; ** $p < 0.05$ *$p < 0.1$.

Figure 1: Screen display of guess game

This figure displays the screen-shot of the guess game, in which a subject guesses the number of white balls in the ambiguous urn. $G_n$ denotes the guess game played after $n$ draws are implemented ($0 \leq n \leq T$, where $T = 14$ in Session I, otherwise $T = 15$). The guess games are played in the following sequence. Guess games are played in line with draws from the ambiguous urn with replacement. The previous draw history, if any, is displayed on the screen for subjects' reference. As an example, the screenshot of the guess game after five draws ($G_5$) is displayed.

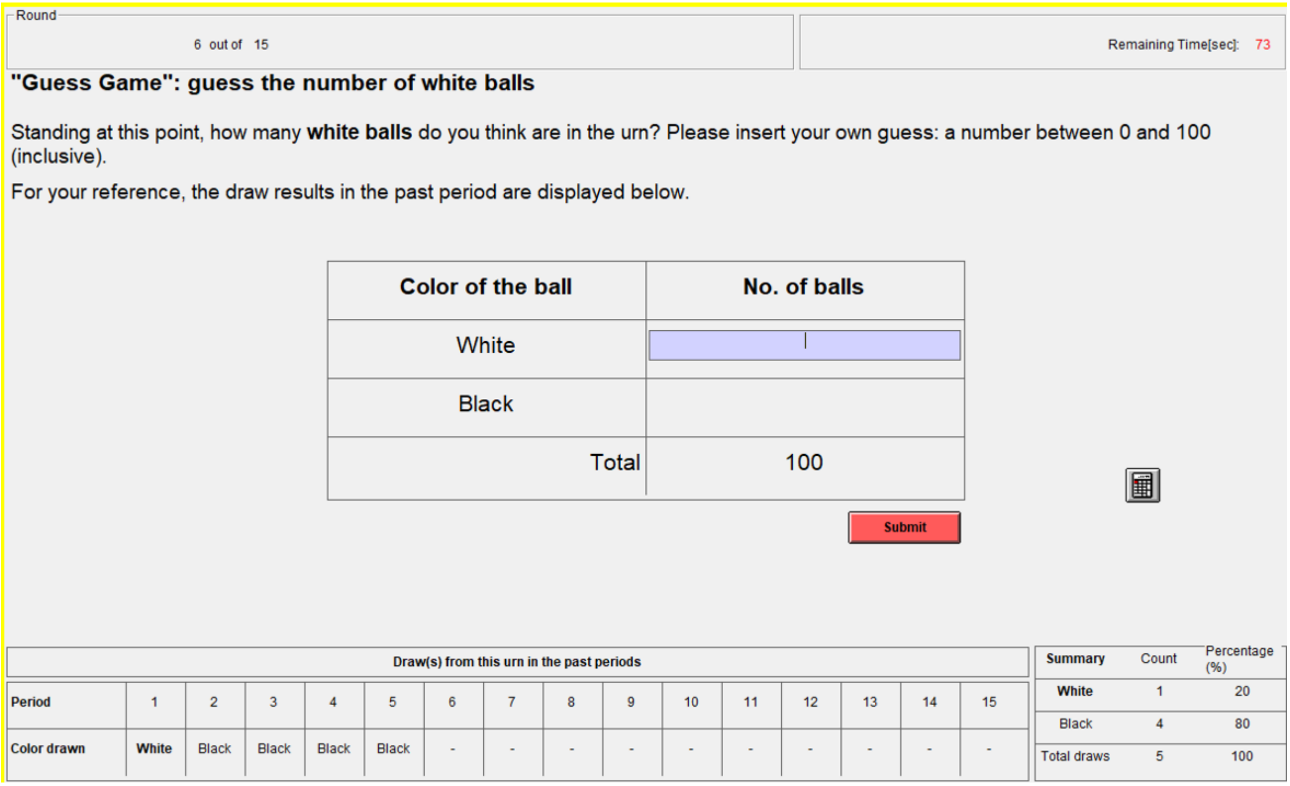
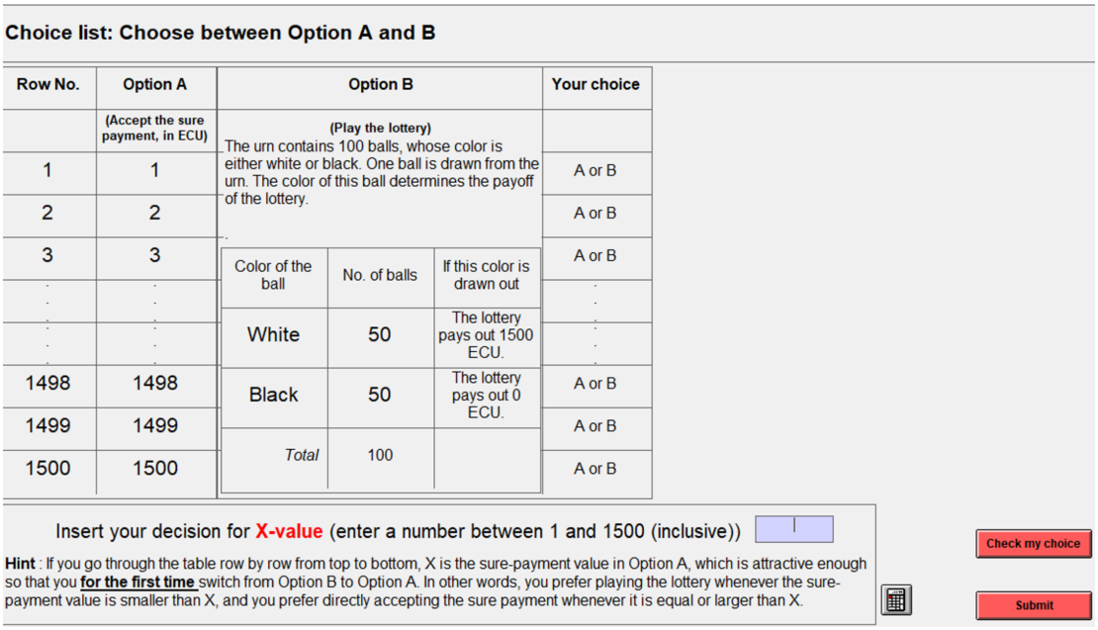


Figure 2: Screen display of choice list

This graph displays the screen-shot of a choice list. As an example, Graph (a) presents risk choice list $R50$: Choose between some sure payment (one of the 1500 versions of Option A) and a 50-50 risky lottery (Option B). Graph (b) presents ambiguity choice list $A_5$: Choose between some sure payment (one of the 1500 versions of Option A) and an ambiguous lottery (Option B), after observing five draws from the ambiguous urn (with replacement). In each choice list, a subject reports a so-called *X-value* to generate 1500 decisions: whenever the sure payment amount in Option A is smaller than this X-value, she prefers playing the lottery in Option B; whenever the sure payment amount in Option A is equal or larger than this X-value, she prefers accepting the sure payment directly.

(a) Risk choice list $R50$




(b) Ambiguity choice list $A_5$

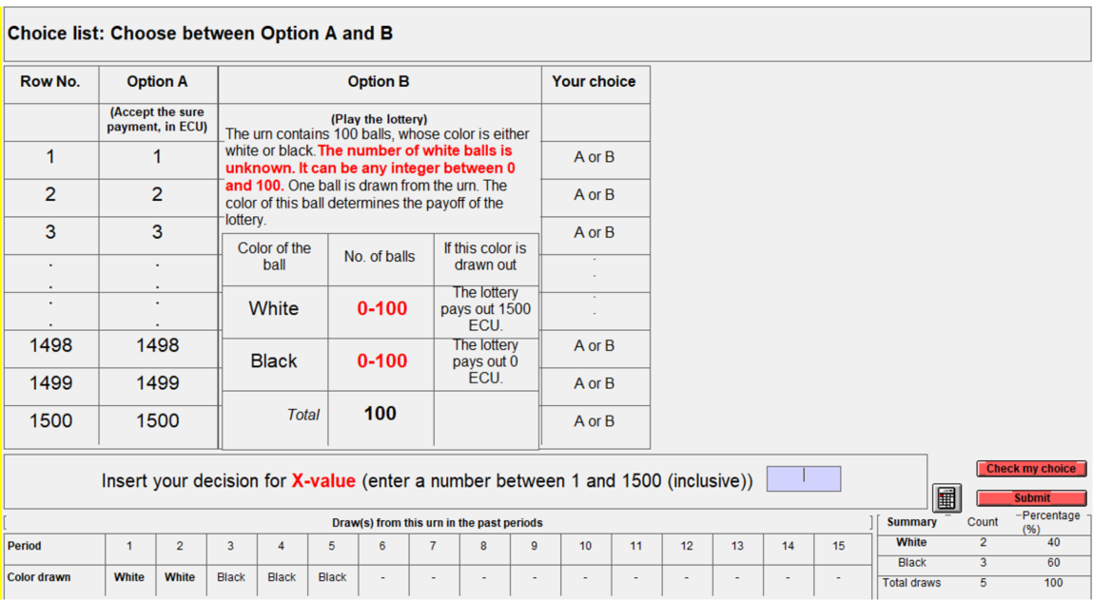


Figure 3: Price deviation and its three components

This diagram illustrates the distributions of the total price deviation, the belief effect, the risk premium, and the ambiguity premium, respectively. For subject $i$, the total price deviation ($TotalDev_{i,n}$) computes the price deviation in comparison with an all-neutral benchmark, when she prices the ambiguous asset in the ambiguity choice list $A_n$ ($n = 0, 15$). It is computed based on Equation (3). The total price deviation is then decomposed into three components: the belief effect (deviation ascribed to personal beliefs, denoted as $BE_{i,n}$), the risk premium (deviation arising from non-neutral risk attitude , denoted as $RP_{i,n}$), and the ambiguity premium (deviation arising from non-neutral ambiguity attitude, $AP_{i,n}$). They are computed based on Equation (5)-(7), respectively. The distribution of each variable (across subjects) is plotted for $n = 0$ (before any draw has occurred) and $n = 15$ (after 15 draws), respectively. Outsiders are not shown in the graph.

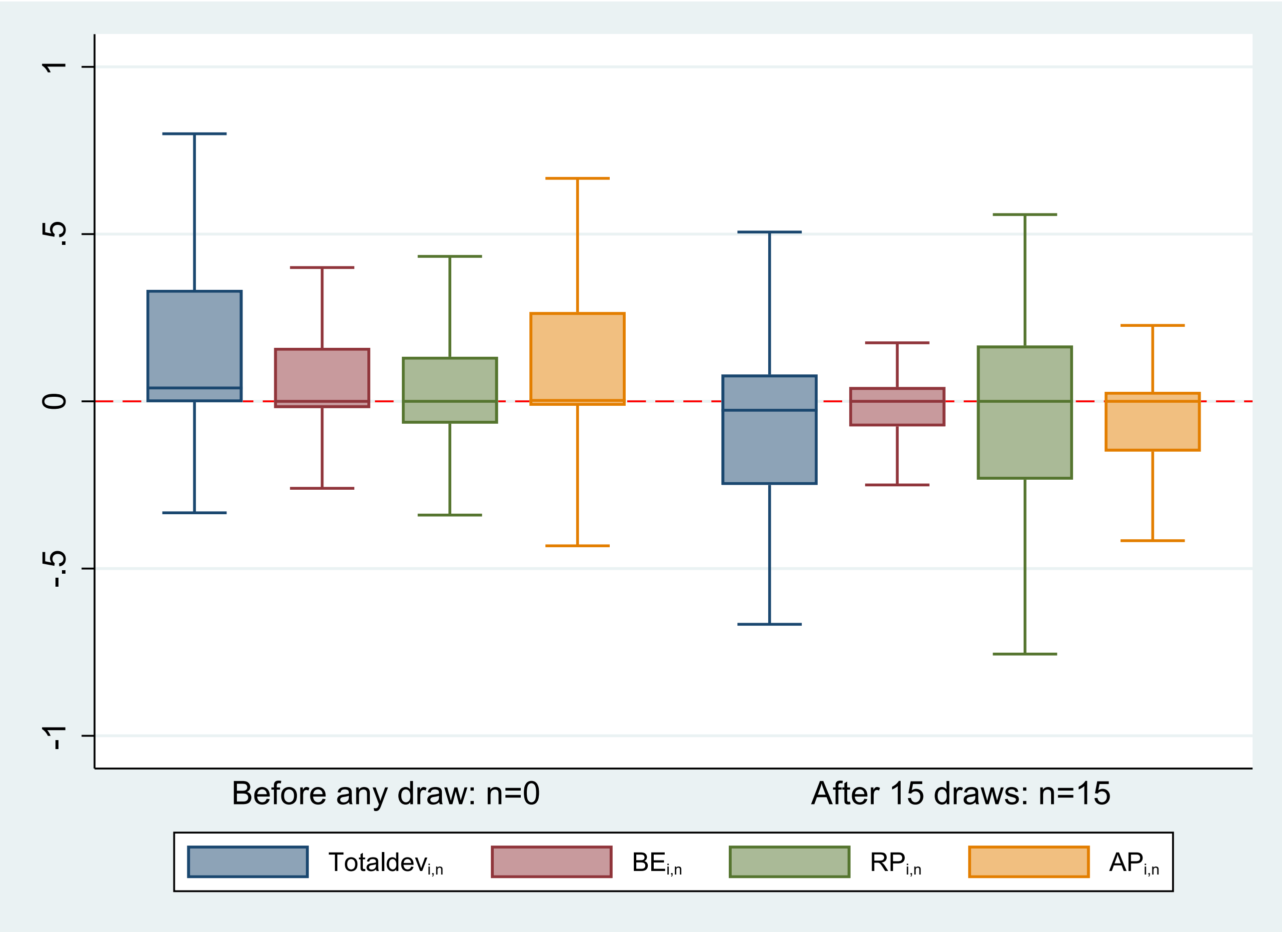
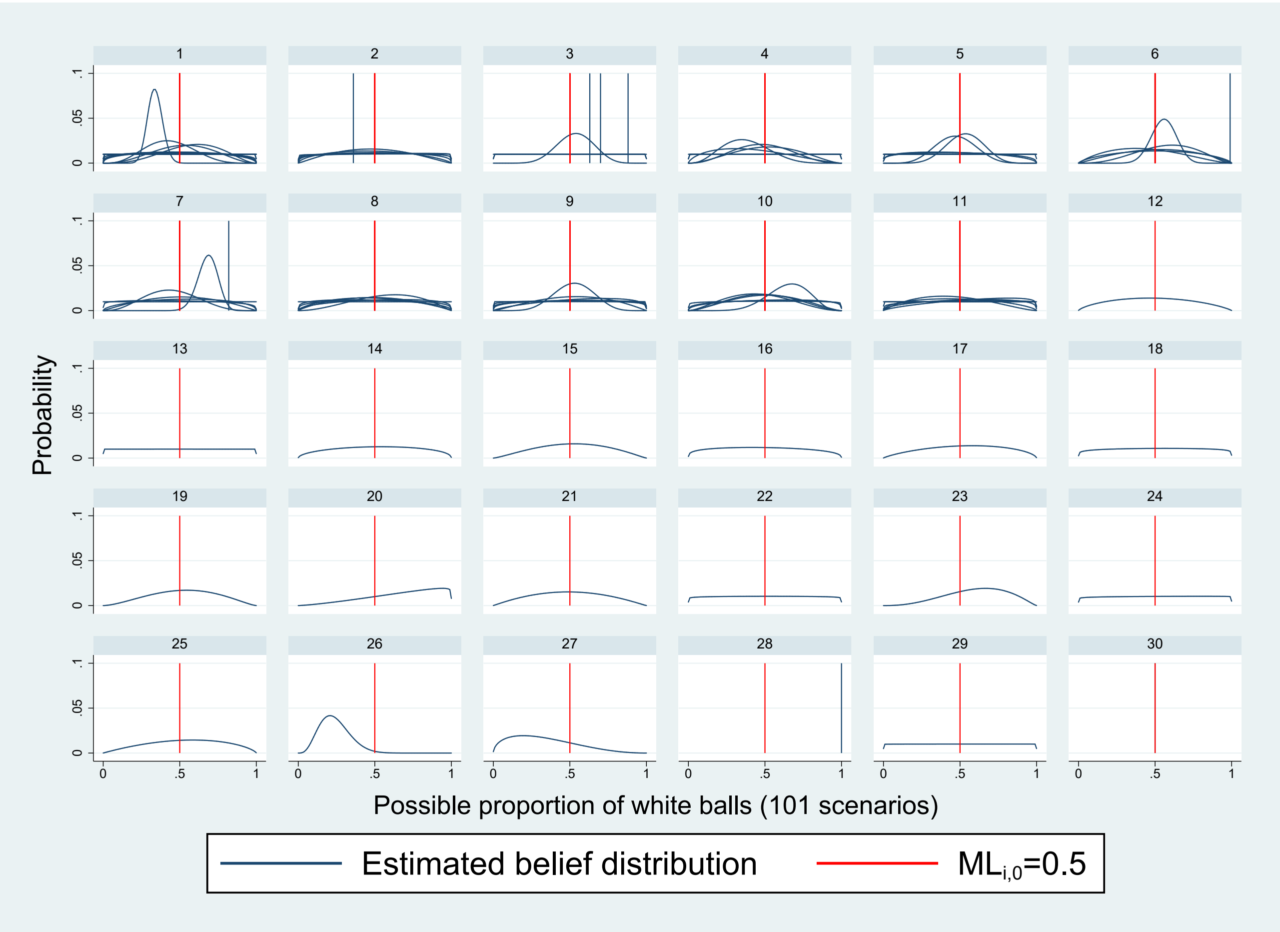


Figure 4: Estimated belief dynamics

This diagram illustrates the belief updating dynamic recovered from estimated parameters $\hat{\alpha}_i$, $\hat{\beta}_i$, $\hat{\gamma}_i^w$ and $\hat{\gamma}_i^b$ of each subject. Only the belief distributions before any draw has occurred ($n = 0$) and after 15 draws ($n = 15$) are illustrated. For better visibility, squeezed distributions with maximum PMF$>$ 0.1 are represented by vertical lines (resting on the support where the maximum PMF is achieved). Subjects are grouped according to the observed draw history (Sub-graphs 1-30 represent 30 different draw history paths): subjects who observe the same draw history are reported in the same sub-graph and are compared with the same benchmark beliefs. The benchmark beliefs of each draw history are added as reference lines (The vertical red lines represent $ML_{i,0} = 0.5$ and $ML_{i,15}$ for each specific draw history.)

(a) Before any draw has occurred ($n = 0$)



(b) After 15 draws ($n = 15$)

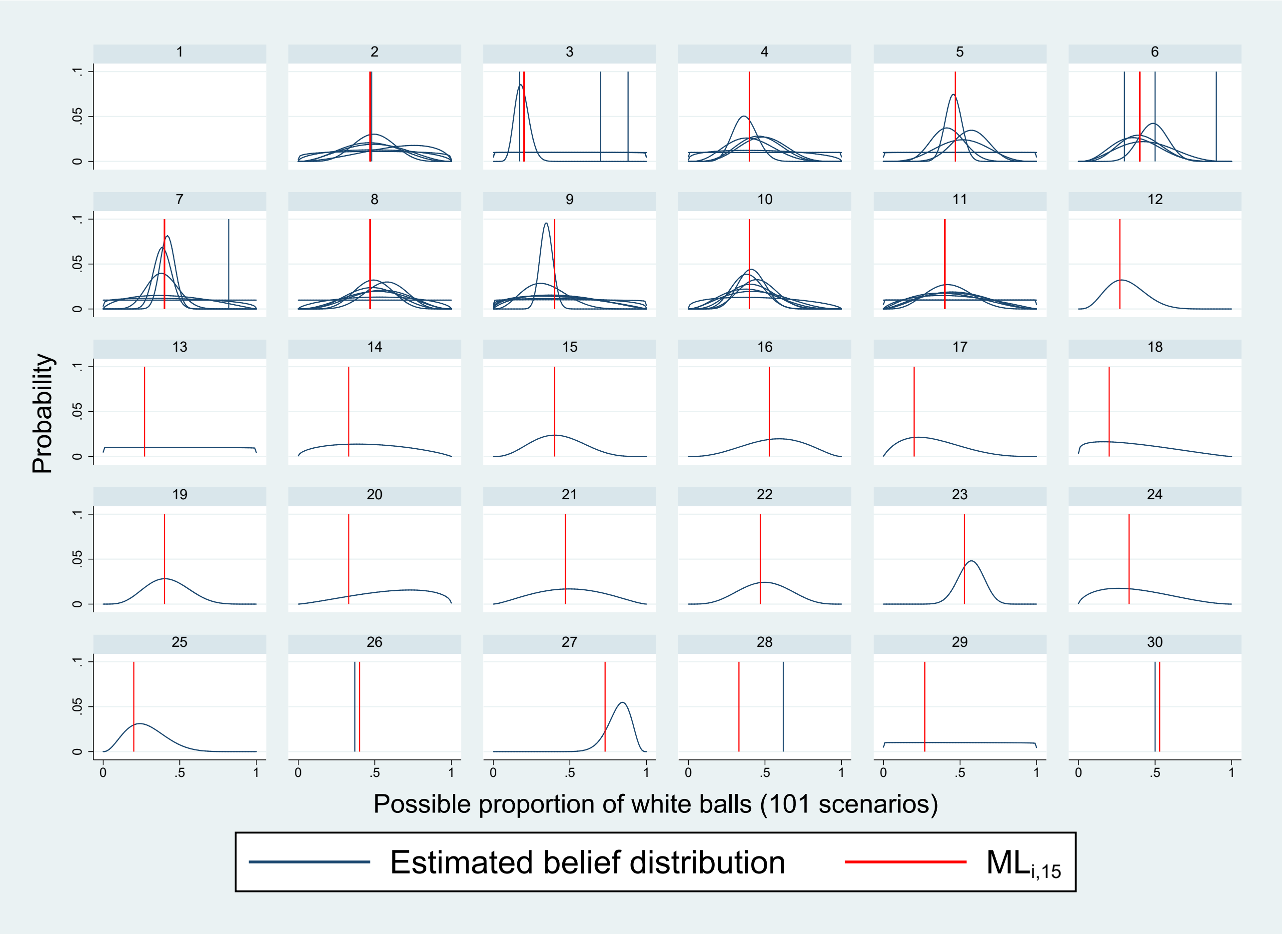
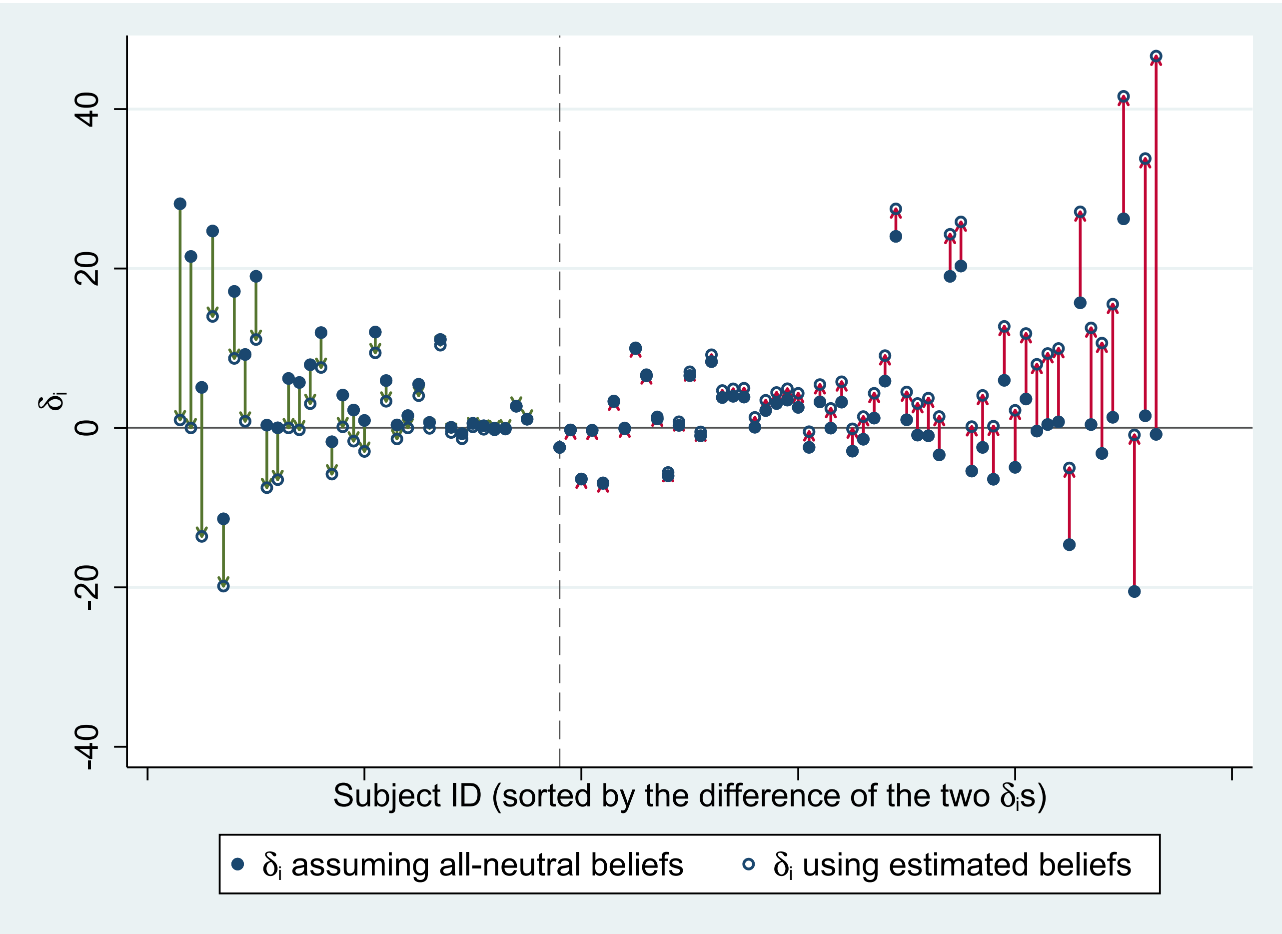


Figure 5: Ambiguity attitude parameter ($\delta_i$): estimations under different utilities and beliefs

This diagram illustrates the estimated ambiguity attitude parameter $\hat{\delta}_i$ under different utility specifications and different belief characterizations. Figure (a) illustrates $\hat{\delta}_i$s estimated based on Specification 1 (CRRA+CRAA). The utility function governing the risk attitude is specified in Equation (27); The utility function governing the ambiguity attitude is specified in Equation (30)-(31). Figure (b) illustrates $\hat{\delta}_i$s estimated based on Specification 2 (CRRA+CAAA). The utility function governing the risk attitude is specified in Equation (27); The utility function governing the ambiguity attitude is specified in Equation (35)-(36). In each Specification, for each subject, a pair of $\hat{\delta}_i$s is estimated under two different beliefs, respectively: either assuming the all-neutral beliefs or using the estimated personal beliefs. For each subject, the two $\hat{\delta}_i$s are illustrated on a vertical line. The difference of the two $\hat{\delta}_i$s is visualized by an arrow pointing from the $\hat{\delta}_i$ assuming all-neutral beliefs to the $\hat{\delta}_i$ using estimated personal beliefs. The illustrations are sorted (on the X-axis) by this difference. The vertical gray dash line indicates where such difference is zero. The green (red) arrows represent the cases in which the $\hat{\delta}_i$ using the estimated personal beliefs is smaller (larger) than the $\hat{\delta}_i$ assuming all-neutral beliefs. For visibility, only $\hat{\delta}_i \in [-50, 50]$ are included.

(a) Specification 1: CRRA and **CRAA** (N=89)



(b) Specification 2: CRRA and **CAAA** (N=91)

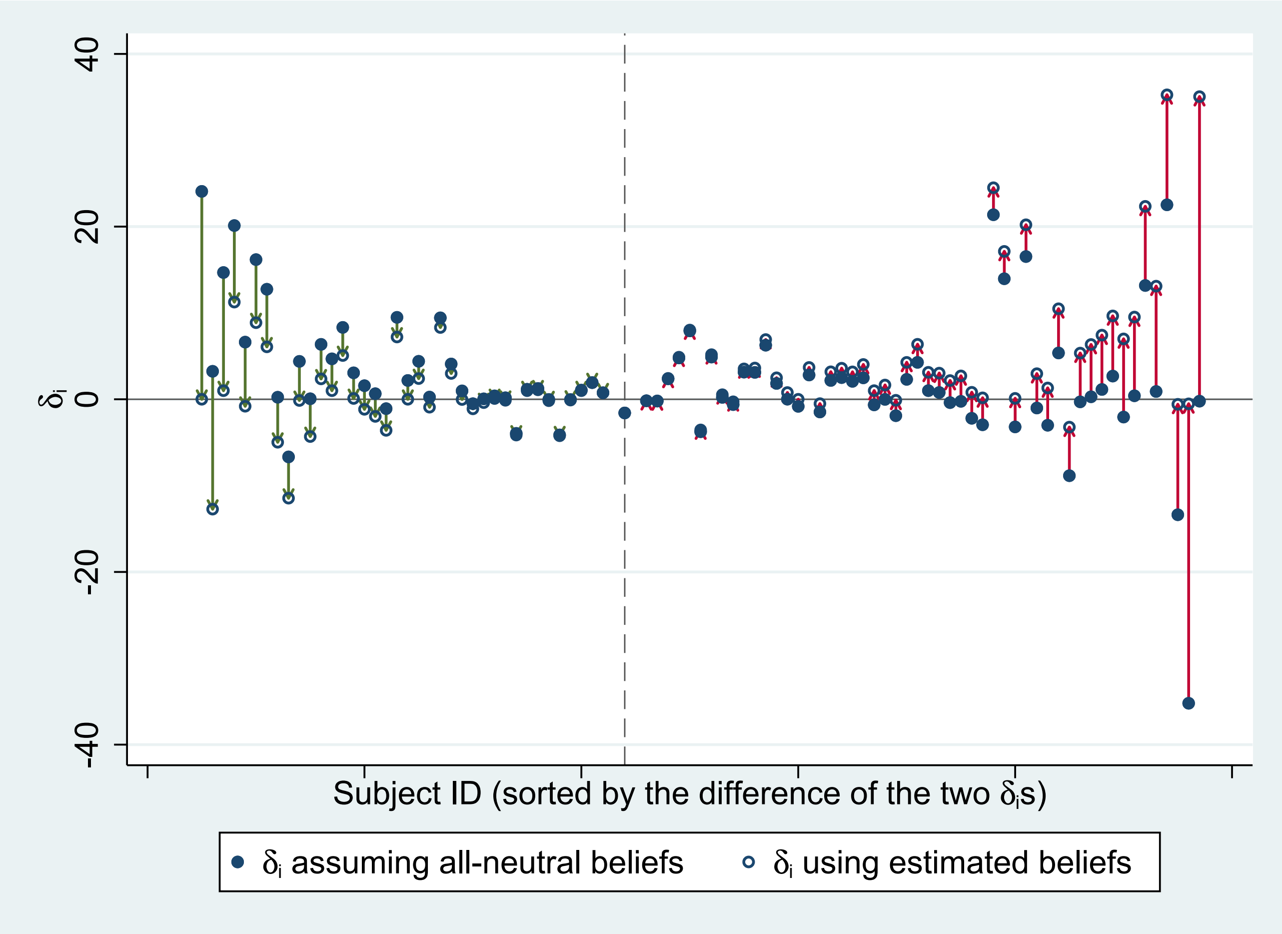
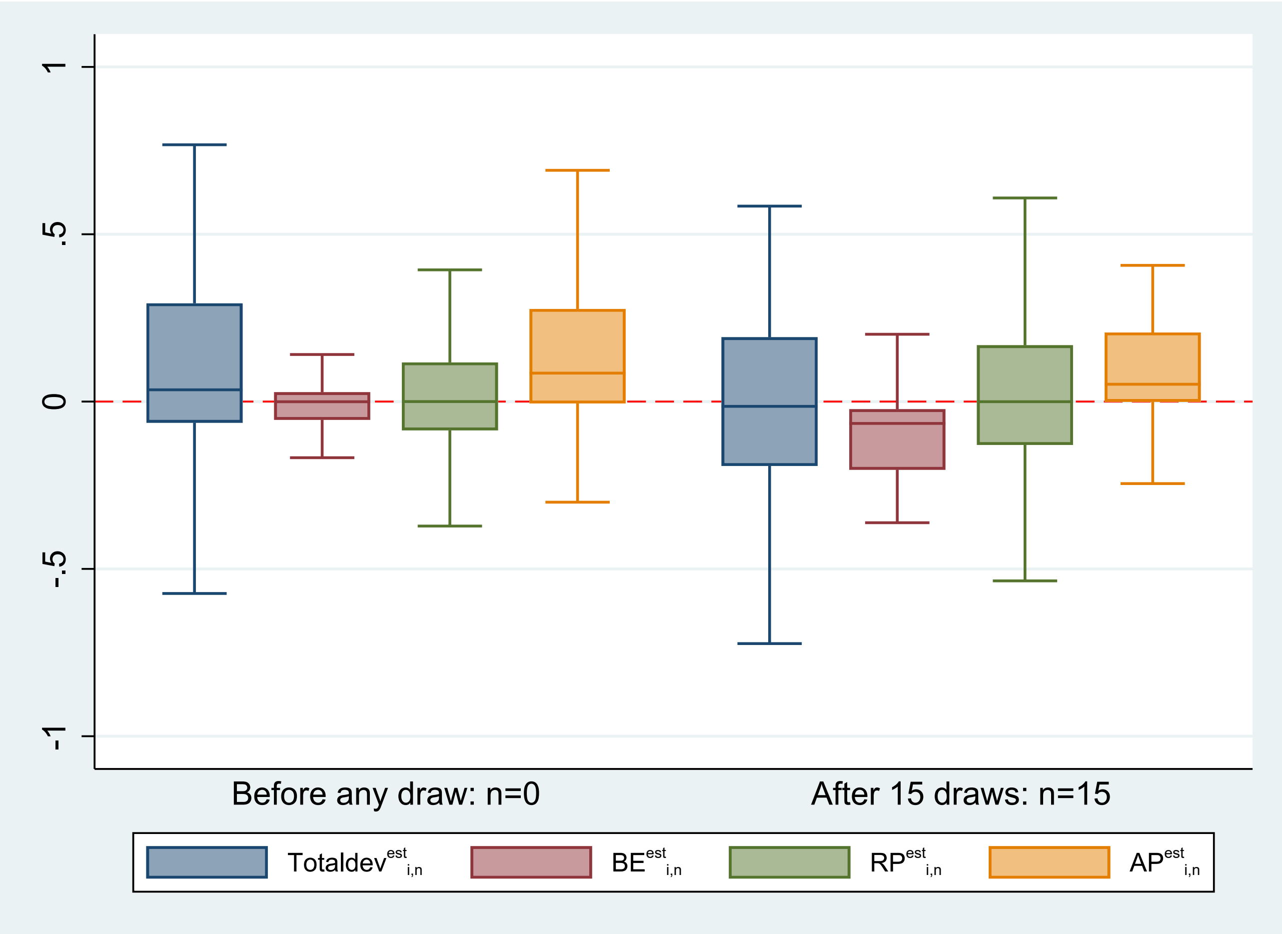


Figure 6: Price deviation and its three components

This diagram illustrates the distributions of the estimated total price deviation, belief effect, risk premium, and ambiguity premium, respectively. All four variables are estimations based on the estimated belief and the estimated attitude parameters. For subject $i$, the estimated total price deviation ($TotalDev_{i,n}^{est}$) computes the price deviation in comparison with an all-neutral benchmark, when subject $i$ prices the ambiguous asset in the ambiguity choice list $A_n$ ($n = 0, 15$). $TotalDev_{i,n}^{est}$ is computed based on Equation (38). The estimated total price deviation is then decomposed into three components: the belief effect (deviation ascribed to personal beliefs, denoted as $BE_{i,n}^{est}$), the risk premium (deviation arising from non-neutral risk attitude , denoted as $RP_{i,n}^{est}$), and the ambiguity premium (deviation arising from non-neutral ambiguity attitude, $AP_{i,n}^{est}$). They are computed based on Equation (41), (42) and (43), respectively. The distribution of each variable (across subjects) is plotted for $n = 0$ (before any draw has occurred) and $n = 15$ (after 15 draws), respectively. Outsiders are not shown in the graph.



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