HOUSE OF FINANCE

WORKING PAPER SERIES

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Bank Networks: Contagion, Systemic Risk and Prudential Policy

SAFE Working Paper No. 87

SAFE | Sustainable Architecture for Finance in Europe

A cooperation of the Center for Financial Studies and Goethe University Frankfurt

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Non-Technical Summary

The propagation of bank losses which turned a shock to a small segment of the US financial system (the sub-prime mortgage market) into a large global banking crisis in 2007/2008 was due to multiple channels of contagion: liquidity hoarding due to banks' precautionary behavior, direct cross-exposures in interbank markets and fire sale externalities. We explore those three channels by focusing on liquidity hoarding and by building an interbank network with risk averse banks who solve non linear portfolio optimization while being linked to other banks in interbank markets and through asset commonality

Our banking network is well in line with empirical facts as it reproduces dis-assortative behavior, core-periphery structure, low density and low clustering coefficients. We asses contagion through different systemic risk metrics, namely centrality measures and Shapley values. We find that indeed banks' risk aversion plays an important role and that liquidity hoarding amplifies losses beyond the ones due to interconnections externalities. Given the realm of our model we test whether different regulatory policy can alleviate contagion. We find that increasing the liquidity requirement unequivocally reduces systemic risk and the contribution of each bank to it. The strong reduction in non-liquid assets induces costs in terms of system efficiency, highlighting the existing trade-off between stability and efficiency. An increase in the equity requirement instead does not present this strong trade-off.

Bank Networks: Contagion, Systemic Risk and Prudential Policy*

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First Draft: January 2014. This Draft: July 2015

Abstract

We present a network model of the interbank market in which optimizing risk averse banks lend to each other and invest in non-liquid assets. Market clearing takes place through a tâtonnement process which yields the equilibrium price, while traded quantities are determined by means of a matching algorithm. Contagion occurs through liquidity hoarding, interbank interlinkages and fire sale externalities. The resulting network configuration exhibits a core-periphery structure, dis-assortative behavior and low density. Within this framework we analyze the effects of prudential policies on the stability/efficiency trade-off. Liquidity requirements unequivocally decrease systemic risk but at the cost of lower efficiency (measured by aggregate investment in non-liquid assets). Equity requirements tend to reduce risk (hence increase stability) without reducing significantly overall investment.

Keywords: *banking networks, systemic risk, contagion, fire sales, prudential regulation.*
JEL: *D85, G21, G28, C63, L14.*

*For helpful comments we thank Michael Gofman, Christoph Roling, Martin Summer, and participants at the Banque de France conference on "Endogenous Financial Networks and Equilibrium Dynamics", Isaac Newton Institute for Mathematical Sciences Workshop "Regulating Systemic Risk: Insights from Mathematical Modeling", Cambridge Center for Risk Studies conference on "Financial Risk and Network Theory", Bundesbank/ESMT/DIW/CFS conference "Achieving Sustainable Financial Stability", European Economic Association Meetings 2014, Bundesbank seminar, Chicago Meeting Society for Economic Measurement 2014, ECB Macro-prudential Research Network Conference, FIRM Research Conference 2014, Unicredit Workshop at Catholic University in Milan "Banking Crises and the Real Economy" and DFG Workshop on "Financial Market Imperfections and Macroeconomic Performance". Parts of this research have been supported by the Frankfurt Institute for Risk Management and Regulation (FIRM). Aldasoro and Faia gratefully acknowledge research support from the Research Center SAFE, funded by the State of Hessen initiative for research LOEWE. Delli Gatti gratefully acknowledges financial support from the FP7 SSH project RASTANEWS (Macro-Risk Assessment and Stabilization Policies with New Early Warning Signals).

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1 Introduction

The propagation of bank losses which turned a shock to a small segment of the US financial system (the sub-prime mortgage market) into a large global banking crisis in 2007-2008 was due to multiple channels of contagion: liquidity hoarding due to banks' precautionary behavior, direct cross-exposures in interbank markets and fire sale externalities. In the face of shocks to one segment of the financial markets and increasing uncertainty, banks start to hoard liquidity. As a result of the market freeze[1], many banks find themselves unable to honor their debt obligations in interbank markets. To cope with liquidity shocks and to fulfill equity requirements, most banks are forced to sell non-liquid assets: the ensuing fall in asset prices[2] produces, under mark-to-market accounting, indirect losses to the balance sheet of banks exposed to those assets. Liquidity spirals turn then into insolvency.

Several papers have shown that credit interlinkages and fire sale externalities are not able to produce large contagion effects if taken in isolation.[3] Our model embeds both channels and envisages a third crucial channel, namely liquidity hoarding. To the best of our knowledge, so far no theoretical model has jointly examined these channels of contagion to assess their impact on systemic risk. After dissecting the qualitative and quantitative aspects of risk transmission, we use the model to determine which prudential policy requirements can strike the best balance between reducing systemic risk and fostering investment in long term assets.

To examine the above channels of contagion and to assess the efficacy of prudential regulation we build a banking network model. The model consists of N *risk averse* heterogeneous banks which perform optimizing portfolio decisions constrained by equity and liquidity requirements. Our framework integrates the micro-foundations of optimizing banks' decisions within a network structure with interacting agents. Indeed, we do not adopt the convention often used in network models according to which links among nodes are exogenous (and probabilistic) and nodes' behavior is best described by heuristic rules. On the contrary, we adopt the well established economic methodology according to which agents are optimizing, decisions are micro-founded and the price mechanism is endogenous.

The convexity in the optimization problem has two implications. First, banks can be both borrowers and lenders at the same time: this is a realistic feature of interbank markets. Second, coupled with convex marginal objectives in profits, it generates precautionary liquidity hoarding in the face of large shocks. The emerging liquidity freeze contributes to exacerbate loss propagation.[4] Banks

[1]The increase in the LIBOR rate was a clear sign of liquidity hoarding. After the sub-prime financial shock the spread between the LIBOR and the U.S. Treasury went up 2% points and remained so for about nine months. As a mean of comparison during the Saving and Loans crisis the spread went up 1% point and remained so for nearly a month.

[2]Fire sales are akin to pecuniary externalities as they work through changes in market prices and operate in the presence of equity constraints. See Greenwood et al. (2015) and Mas-Colell et al. (1995), chapter 11.

[3]See for instance Caccioli et al. (2014) or Glasserman and Young (2014).

[4]See also Afonso and Shin (2011).

invest in non-liquid assets, which trade at common prices, hence fire sale externalities emerge. Our banks also trade debt contracts with each other in the interbank market, hence defaults and debt interlinkages contribute to loss propagation. Markets are defined by a price vector and a procedure to match trading partners. The equilibrium price vector (in both the interbank and non-liquid asset markets) is reached through a tâtonnement process,[5] in which prices are endogenously determined by sequential convergence of excess demand and supply. Once prices are determined, actual trading among heterogeneous banks takes place through a matching algorithm (see Gale and Shapley (1962) and Shapley and Shubik (1972)). To match trading partners in the interbank market we use a *closest matching* (or *minimum distance*) algorithm. Before examining the contagion channels in our model we assess its empirical performance and find that it can replicate important structural/topological features of real world interbank networks (core-periphery structure, low density, dis-assortative behavior)[6].

In assessing the contagion channels we find a strong connection between the contribution of banks to systemic risk and their total assets.[7] When considering specific balance sheet items, we find that both high interbank borrowing as well as high investment in non-liquid assets are important in explaining the contribution of banks to systemic risk generation. High interbank borrowing increases the scope of risk transmission through direct debt linkages. Investment in non-liquid assets enlarges the scope of fire sale externalities. Both channels are amplified if we take into account risk averse banks. When we analyze the impact of regulatory policy interestingly we find that an increase in the liquidity requirement reduces systemic risk more sharply and more rapidly than an increase in equity requirements. As banks are required to hold more liquidity, they reduce their exposure in the interbank market as well as their investment in non-liquid assets in absolute terms. The fall in interbank supply produces an increase in the interbank interest rate, which, due to asset substitution, induces a fall in non-liquid asset investment relative to interbank lending. Banks become less interconnected in the interbank market and less exposed to swings in the price of non-liquid assets. Both channels of contagion (cross-exposures and fire sale externalities) become less active. With an increase in the equity requirement instead the demand of interbank borrowing falls and so does the interbank rate. Banks substitute interbank lending, which has become less profitable, with investment in non-liquid assets. While the scope of network externalities and cascades in debt defaults falls, the scope of pecuniary externalities increases. On balance systemic risk, and the contribution of each bank to it, declines, but less than with an increase in liquidity requirements.

[5]See also Cifuentes et al. (2005), Bluhm et al. (2014), Duffie and Zhu (2011).

[6]For a recent summary including further references see Langfield and Soramäki (2014).

[7]Systemic risk is measured by the aggregate probability of default in the system and banks' contribution to it by means of the Shapley value. The latter has been borrowed from the literature on both cooperative and non-cooperative games. See Shapley (1953) and Gul (1989) respectively for the seminal contributions, and Drehmann and Tarashev (2013) and Bluhm et al. (2014) for applications to banking. In particular, we follow closely the latter. Other centrality measures for systemic importance are considered in one of the appendices.

The rest of the paper is structured as follows. Section 2 relates our paper to the literature. Section 3 describes the model. Section 4 presents the baseline network topology and discusses the empirical matching. Section 5 analyzes the response of the network model to shocks and the contribution of each bank to systemic risk. Section 6 focuses on the policy analysis. Section 7 concludes. Appendices with figures and tables follow.

2 Related Literature

After the collapse of Lehman and the worldwide spreading of financial distress two views have emerged regarding the mechanisms triggering contagion.

According to the first one, cascading defaults are due to credit interconnections. In high value payment systems banks rely on incoming funds to honor payments of outflows; when synchronicity breaks down and banks fail to honor debts, cascading defaults emerge. Eisenberg and Noe (2001), Afonso and Shin (2011) or Elliott et al. (2014) analyze this channel using lattice-theoretic methods to solve for the unique fixed point of an equilibrium mapping. Works in this area take the payment relations as given; we make a step forward as credit interlinkages in our model result from portfolio optimization and endogenous price mechanisms, in the spirit of recent contributions like Bluhm et al. (2014) and Halaj and Kok (2015), among others.

According to the second view, financial distress is triggered by fire sale externalities in environments characterized by asset commonality coupled with mark-to-market accounting and equity requirements (see also Greenwood et al. (2015)). As one bank is hit by a shock, it tries to sell assets to meet VaR or capital constraints. Under mark-to-market accounting, the endogenous fall in market prices negatively affects other banks' balance sheets. Cifuentes et al. (2005) formalized this mechanism, which was subsequently used by Bluhm et al. (2014) among others. In particular, our paper builds on the latter contribution.

Our model encompasses both views and shows that both are important to account for risk propagation. Moreover, we bring to the fore a third mechanism based on liquidity hoarding: once financial distress has emerged banks become more cautious and hoard liquidity. The ensuing liquidity freeze amplifies risk propagation. A similar channel is present also in Afonso and Shin (2011).

Our paper is also related to three other strands of recent literature. First, it contributes to the literature which tries to assess the trade-offs between risk sharing and risk propagation. Using an interbank network, Allen and Gale (2000) show the existence of a monotonically decreasing relation between systemic risk and the degree of connectivity.[8] More recent views challenge - at least in part - this conclusion by showing that a *trade off* emerges between decreasing individual risk due

[8] In their model each bank is linked only to one neighbor along a ring. They show that the probability of a bankruptcy avalanche is equal to one in the credit chain, but that, as the number of partners of each bank increases (namely when the credit network becomes complete), the risk of individual default goes asymptotically to zero due to the improved risk sharing possibilities.

to risk sharing and increasing systemic risk due to the amplification of financial distress. Battiston et al. (2012) show for instance that the relation between connectivity and systemic risk is hump shaped: at relatively low levels of connectivity, the risk of individual default goes down with density thanks to risk sharing while at high levels of connectivity, a positive feedback loop makes a bank under distress more prone to default as the number of partners under distress increases.[9] In the numerical simulations of our model, we will assume a multinomial distribution of correlated shocks in order to capture the presence of feedback loops.

Secondly, our paper is related to the literature analyzing metrics of systemic risk and measuring the contribution of each bank to it (namely metrics of systemic importance). Third, a connection can also be established with the literature analyzing matching mechanisms in markets along the lines indicated by Shapley and Shubik (see for instance Shapley and Shubik (1972)). Finally, our paper is related to an emerging literature studying prudential regulation in financial networks (see for instance Gai et al. (2011) among many others).

3 The Banking Network

At a general level, a network can be represented by a list of nodes and the links connecting them. When applied to banking, it is straightforward to identify the nodes with banks and the links with the borrowing and lending relationships between the banks. In this spirit, the interbank system can be succintly summarized by a matrix $\mathbf{X}$ with element x_{ij} representing the exposure (through lending) of bank i to bank j. We consider a financial system consisting of N banks, hence the matrix $\mathbf{X}$ will be of dimension $n \times n$. Two important features of our network are worth noting: (i) it is a weighted network, i.e. a link between banks i and j is indicated by the element $x_{ij} \in \mathbb{R}_{\geq 0}$ and represents the amount (in money) lent by bank i to bank j; (ii) it is a directed network, i.e. the existence of a link in one direction does not imply the existence of a link going in the opposite direction and therefore the matrix is not necessarily symmetric ($x_{ij} \neq x_{ji},\ i \neq j$). Notice that each bank can be both a borrower and a lender vis-à-vis different counterparties. An important aspect is that cross-lending positions (hence the network links) result endogenously from the banks' optimizing decisions (see next section) and the markets' tâtonnement processes. Banks in our model are characterized also by external (non interbank) assets (cash and non-liquid assets) and liabilities (deposits). As usual, equity or net worth is defined as the difference between total assets and total liabilities. By assumption, banks are heterogeneous due to different returns on non-liquid assets and the levels of calibrated equity and deposits.

Prices in the interbank market and the market for non-liquid assets are determined by tâtonnement processes. In setting up the benchmark banking system the interbank tâtonnement process is instrumental in delivering interbank market equilibrium, whereas after setting the system and in

[9] Also Gai et al. (2011) derive a non-monotonic relationship between connectivity and systemic risk.

the aftermath of a shock the tâtonnement process in the market for non-liquid assets captures the unfolding of fire sales and is instrumental in the amplification of the shock tranmission process. The logic of the tâtonnement processes implies the introduction of fictitious Walrasian auctioneers (see also Cifuentes et al. (2005) or Duffie and Zhu (2011)) which collect individual notional quantities, aggregate them and adjust the relevant price in order to bring the notional aggregate demand and supply in line with each other.[10] Once a clearing price has been achieved, actual trade takes place. Traded quantities in the interbank market are determined according to a closest matching algorithm (see Section 3.2 for details). A general overview of the model and the channels which operate in it are described visually in Appendix B.

3.1 The banking problem

Our network consists of optimizing banks which solve portfolio optimization problems subject to regulatory and balance sheet constraints. Banks are risk averse and have convex marginal utilities. The convex optimization problem (concave objective function subject to linear constraints) allows us to account for interior solutions for both borrowing and lending. Banks are therefore on both sides of the interbank market vis-à-vis different counterparties: this is a realistic feature of interbank markets and is a necessary condition for a core-periphery configuration to emerge (see Craig and von Peter (2014)). Furthermore we assume that banks have convex marginal utilities with respect to profits.[11] Empirical observation shows that banks tend to adopt precautionary behavior in an uncertain environment.[12] Convex marginal utilities allow us to account for this fact, since in this case banks' expected marginal utility (hence banks' precautionary savings) tends to increase with the degree of uncertainty.

Banks' portfolios are made up of cash, non-liquid assets and interbank lending. Moreover, banks are funded by means of deposits and interbank loans. Hence, the balance sheet of bank i is given by:

$$c_i + pn_i + \underbrace{l_{i1} + l_{i2} + ... + l_{ik}}_{\equiv l_i} = d_i + \underbrace{b_{i1} + b_{i2} + ... + b_{ik'}}_{\equiv b_i} + e_i \tag{1}$$

where c_i represents cash holdings, n_i denotes the volume and p the price of non liquid assets (so that pn_i is the market value of the non liquid portion of the bank's portfolio), d_i stands for deposits and e_i for equity. l_{ij} is the amount lent to bank j where $j = 1, 2, ..., k$ and k is the cardinality of the set of borrowers from the bank in question; b_{ij} is the amount borrowed from bank j where $j = 1, 2, ..., k'$ and k' is the cardinality of the set of lenders to the bank in question.

[10]Banks in our model are risk averse, hence have concave objective functions and linear constraints. The convexity of the optimization problem and the assumption of an exponential aggregate supply function guarantees that individual and aggregate excess demand and supply behave in both markets according to Liapunov convergence.

[11]This amounts to assuming a positive third derivative.

[12]See also Afonso and Shin (2011).

Hence $l_i = \sum_{j=1}^{k} l_{ij}$ stands for total interbank lending and $b_i = \sum_{j=1}^{k'} b_{ij}$ stands for total interbank borrowing.[13]

The bank's optimization decisions are subject to two standard regulatory requirements:

$$c_i \geq \alpha d_i \tag{2}$$

$$\frac{c_i + pn_i + l_i - d_i - b_i}{\omega_n pn_i + \omega_l l_i} \geq \gamma + \tau \tag{3}$$

Equation 2 is a liquidity requirement according to which banks must hold at least a fraction α of their deposits in cash.[14] Equation 3 is an equity requirement (which could also be rationalized as resulting from a VaR internal model). It states that the ratio of equity at market prices (at the numerator) over risk weighted assets (at the denominator) must not fall below a threshold $\gamma + \tau$. Cash enters the constraint with zero risk weight since it is riskless in our model, while ω_n and ω_l represent the risk weights on non-liquid assets and interbank lending respectively. The parameter γ is set by the regulator, while the parameter τ captures an additional desired equity buffer that banks choose to hold for precautionary motives.

The bank's preferences are represented by a CRRA utility function:

$$U(\pi_i) = \frac{(\pi_i)^{1-\sigma}}{1-\sigma} \tag{4}$$

where σ stands for the bank's risk aversion. As explained above the convex maximization problem serves a dual purpose. First, it allows us to obtain interior solutions for borrowing and lending. Second, since the CRRA utility function is characterized by convex marginal utilities (positive third derivatives), we can introduce banks' precautionary behavior in the model. As marginal utilities are convex with respect to profits, higher uncertainty induces higher expected marginal utility at the optimal point. As expected marginal utility increases banks tend to be more cautious and to hoard liquidity more.

Another important aspect of concave optimization is that in non-linear set-ups, the variance in assets' returns affects the bank's decision. Higher variance in assets' returns reduces expected banks' utility, thereby reducing the extent of their involvement both in lending as well non-liquid assets investment. This is also the sense in which higher uncertainty in assets' returns (interbank lending as well as non-liquid assets) produces liquidity hoarding and credit crunches. In this set up it is convenient to take a second order Taylor approximation of the expected utility of profits.

The second order approximation of Equation 4, in the neighborhood of the expected value of

[13] Note that since banks cannot lend to nor borrow from themselves, we set $l_{ii} = b_{ii} = 0 \; \forall \; i = 1, ..., N$.

[14] Basel III proposes the liquidity coverage ratio (LCR), which is somewhat more involved than Equation 2. Given the stylized nature of our model the LCR is not easy to capture, yet we consider that the liquidity requirement in Equation 2 provides a good approximation to the constraints faced by the bank in terms of liquidity management.

profits $E[\pi]$ reads as follows[15]:

$$U(\pi_i) \approx U(E[\pi_i]) + U_\pi(\pi_i - E[\pi_i]) + \frac{1}{2}U_{\pi\pi}(\pi_i - E[\pi_i])^2 \tag{5}$$

Taking expectations on both sides of equation 5 and yields:

$$\begin{aligned} E\left[U(\pi_i)\right] &\approx \underbrace{E\left[U(E[\pi_i])\right]}_{=U(E[\pi_i]) \text{ by LIE}} + U_\pi \underbrace{E\left[(\pi_i - E[\pi_i])\right]}_{=0 \text{ by LIE}} + \frac{1}{2}U_{\pi\pi}\underbrace{E\left[(\pi_i - E[\pi_i])^2\right]}_{=\text{Var}(\pi_i)=\sigma_\pi^2} \\ &\approx U(E[\pi_i]) + \frac{1}{2}U_{\pi\pi}\sigma_\pi^2 \end{aligned} \tag{6}$$

where we have used the law of iterated expectations and where σ_π^2 stands for the variance of profits.

Given the CRRA function $U(\pi_i) = \frac{(\pi_i)^{1-\sigma}}{1-\sigma}$, where σ is the coefficient of risk aversion, we can compute the second derivative as $U_{\pi\pi} = -\sigma E[\pi_i]^{-(1+\sigma)}$. Notice that under certainty equivalence (namely when $E[U'''(\pi)] = 0$) the equality $E\left[U(\pi_i)\right] = U(E[\pi_i])$ holds at all states. With CRRA utility, the third derivative with respect to profits is positive, which in turn implies that the expected marginal utility grows with the variability of profits. Furthermore since, $U'' < 0$, expected utility is equal to the utility of expected profits minus a term that depends on the volatility of bank profits and the risk aversion parameter. This is a direct consequence of Jensen's inequality and provides the standard rationale for precautionary saving. Using the expression derived above for $U_{\pi\pi}$, the expected utility of profits can be written as:

$$E\left[U(\pi_i)\right] \approx \frac{E[\pi_i]^{1-\sigma}}{1-\sigma} - \frac{\sigma}{2}E[\pi_i]^{-(1+\sigma)}\sigma_\pi^2 \tag{7}$$

Equation 7 represents the objective function that bank i maximizes subject to the constraints introduced above. With these elements in mind the problem of bank i can be summarized as follows:

$$\begin{aligned} &\underset{\{c_i,n_i,l_i,b_i\}}{\text{Max}} \quad E[U(\pi_i)] \\ &s.t. \ \ \textit{Equation 2}, \textit{Equation 3}, \textit{Equation 1} \\ &c_i, n_i, l_i, b_i \geq 0 \end{aligned} \tag{P}$$

Before moving forward and for the sake of completeness we derive next the precise form of profits, as well as their variance.

[15] Note that all partial derivatives are also evaluated at $E[\pi]$.

The bank's profits are given by the returns on lending in the interbank market (at the interest rate r^l) plus returns from investments in non-liquid assets (whose rate of return is r_i^n) minus the expected costs from interbank borrowing.[16] The rate of return on non-liquid assets is exogenous and heterogeneous across banks: we assume that banks have access to investment opportunities with different degrees of profitability. The interest rates on borrowed funds are also heterogeneous across banks due to a risk premium.[17] In lending to j, bank i charges a premium r_j^p over the risk-free interest rate (i.e. the interest rate on interbank loans r^l), which depends on the probability of default of j, δ_j. The premium can be derived through an arbitrage condition. By lending l_{ij} to j, bank i expects to earn an amount given by the following equation:

$$\underbrace{(1-\delta_j)\left(r^l+r_j^p\right)l_{ij}}_{\text{with no default}}+\underbrace{\delta_j\left(r^l+r_j^p\right)(1-\xi)\,l_{ij}}_{\text{with default}} \tag{8}$$

where ξ is the loss given default parameter. If bank j cannot default, bank i gets:

$$l_{ij}r^l \tag{9}$$

By equating 8 and 9 we can solve for the fair risk premium charged to counterparty j:

$$r_j^p=\frac{\xi\delta_j}{1-\xi\delta_j}r^l \tag{10}$$

It is immediate to verify that the premium is calculated so that, by lending to j, bank i expects to get $r^l l_{ij}$ (to obtain this, substitute the premium back into 8). We can interpret condition 8 also as a participation constraint: bank i will lend to bank j only if it gets an expected return from lending equal to the risk free rate, i.e. the opportunity cost of lending. By summing up over all possible counterparties of bank i, and recalling that $l_i=\sum_{j=1}^k l_{ij}$, we retrieve the overall gain that bank i expects to achieve by lending to all the borrowers: $r^l l_i$. On the other hand, as a borrower, bank i must also pay the premium associated to its own default probability. Since banks charge a fair risk premium, the returns that banks obtain from non-defaulting borrowers offset the losses resulting from contracts with defaulting borrowers. Borrowing banks, on the other hand, must always pay the premium.[18] Therefore the cost of borrowing is given by: $r_i^b b_i=(r^l+r_i^p)b_i=\frac{1}{1-\xi\delta_i}r^l b_i$.

Finally, the gains from investment in non-liquid assets are given by: $r_i^n\frac{n_i}{p}$. Given these assumptions, the profits of bank i read as follows:

[16] For simplicity it is assumed that deposits and cash/reserves are not remunerated. Note that since these would be a fixed number if calibrated they would only shift up or down the responses that we see from the model. Furthermore, such shifts would be indeed hard to even perceive.

[17] In what follows for the derivation of the premium we draw on Bluhm et al. (2014).

[18] A valid question which may arise in our context regards the validity of the Modigliani-Miller theorem in the framework we present here. In Appendix A we briefly show why this theorem does not hold in our setting.

$$\pi_i = r_i^n \frac{n_i}{p} + r^l l_i - (r^l + r_i^p) b_i = r_i^n \frac{n_i}{p} + r^l l_i - \frac{1}{1-\xi\delta_i} r^l b_i \tag{11}$$

Having obtained an expression for profits, we now compute their variance. Notice that volatility only derives from uncertainty in non-liquid asset returns and from default premia on borrowing. These are cross-sectional variances and they are the only which can be considered in our setting, which is static and hence does not allow for the consideration of time series variances. The return on interbank lending as well as the price of non-liquid assets are endogenous and therefore will ultimately depend on exogenous elements of the model and of the shocks assumed.[19] Finally, it should be noted that in setting up the system the price of non-liquid assets is set to one, which is a status-quo scenario in which aggregate sales of non-liquid assets are zero and therefore no fire sales are present. Given the sources of uncertainty we obtain the following volatility of profits:

$$\sigma_\pi^2 = \text{Var}\left(r_i^n \frac{n_i}{p} + r^l l_i - \frac{1}{1-\xi\delta_i} r^l b_i \right) \tag{12}$$

$$= \left(\frac{n_i}{p}\right)^2 \sigma_{r_i^n}^2 - (b_i r^l)^2 \text{Var}\left(\frac{1}{1-\xi\delta_i}\right) + 2 n_i r^l b_i \text{cov}\left(r_i^n, \frac{1}{1-\xi\delta_i}\right) \tag{13}$$

We know that $\delta_i \in [0,1]$. Furthermore, even when $f(\delta_i) = \frac{1}{1-\xi\delta_i}$ is a a convex function, over a realistic range of δ_i it is essentially linear and it is therefore sensible to obtain the variance of $f(\delta_i)$ through a first order Taylor approximation around the expected value of δ_i, which yields:

$$\text{Var}\left(\frac{1}{1-\xi\delta_i}\right) = \xi^2 (1 - \xi E[\delta_i])^{-4} \sigma_{\delta_i}^2 \tag{14}$$

We assume that the ex ante correlation between return on non-liquid assets and costs of borrowing is zero, hence we can set the covariance term in Equation 12 to zero. This leaves us with the following expression for the variance of profits:

$$\sigma_\pi^2 = \left(\frac{n_i}{p}\right)^2 \sigma_{r_i^n}^2 - (b_i r^l)^2 \xi^2 (1 - \xi E[\delta_i])^{-4} \sigma_{\delta_i}^2 \tag{15}$$

3.2 Interbank Market Clearing

The interbank market clears in two stages. In the first stage a standard tâtonnement process is applied and the interbank interest rate is obtained by clearing excess demand/supply. Individual demands and supplies (as obtained from banks' optimization) are summed up to obtain market

[19] Furthermore, given the nature of the fire sales externalities, it is virtually impossible for banks to form an expectation about them, as they would need to know the entire balance sheet of the banking system in every state of the world. For a similar argument see Caballero and Simsek (2013).

demand and supply. If excess demand or supply occurs at the market level, the interbank rate is adjusted sequentially to eliminate the discrepancy. In the second stage, after the equilibrium interbank rate has been determined, a matching algorithm determines the actual pairs of banks involved into bilateral trading (at market prices). We aim to capture here the behavior of centralized interbank markets as opposed to markets in which bilateral bargaining is the main mechanism driving the matching of banks.[20] Additionally, as noted by Glasserman and Young (2014), to assess the potential damage that can come from interbank connections the precise shape of the network is not as important as some balance sheet ratios that better capture this potential damage, like for instance total interbank borrowing or total assets/liabilities. These are precisely the quantities on which banks focus in our model, as we aim to assess how banks navigate the trade-offs between the different types of externalities and their investment in long term assets.

Price Tâtonnement in the Interbank Market. For a given calibration of the model, which includes an initial level of the interbank interest rate, the bank chooses the optimal demand (b_i) and supply (l_i) of interbank debt trading. These are submitted to a Walrasian auctioneer who sums them up and obtains the market demand $B = \sum_{i=1}^{N} b_i$ and supply $L = \sum_{i=1}^{N} l_i$. If $B > L$ there is excess notional demand in the market and therefore r^l is increased, whereas the opposite happens if $B < L$.[21] Changes in the interbank rates are bounded within intervals which guarantee the existence of an equilibrium see Mas-Colell et al. (1995)).

The clearing price process delivers an equilibrium interest rate as well as two vectors, $\mathbf{l} = [l_1\ l_2\ ...\ l_N]$ and $\mathbf{b} = [b_1\ b_2\ ...\ b_N]$, which correspond to optimal lending and borrowing of all banks for given equilibrium prices.

Matching Trading Partners. Once the equilibrium interest rate has been obtained, actual bilateral trading relations among banks need to be determined. In other words, given the vectors $\mathbf{l} = [l_1\ l_2\ ...\ l_N]$ and $\mathbf{b} = [b_1\ b_2\ ...\ b_N]$ obtained during the price clearing process we need to match pairs of banks for the actual trading to take place. We use a matching algorithm to determine how bank i distributes its lending ($l_i = \sum_{i=1}^{k} l_{ij}$) and/or borrowing ($b_i = \sum_{i=1}^{k'} b_{ij}$) among its potential counterparties.

The matching algorithm, therefore, will determine the structure of the network. Mathematically the matching algorithm delivers the matrix of interbank positions $\mathbf{X}$, with element x_{ij} indicating the exposure (through lending) of bank i to bank j, starting from the vectors $\mathbf{l}$ and $\mathbf{b}$. Once all trading has been cleared the vectors $\mathbf{l}$ and $\mathbf{b}$ will also correspond to the row sum and column sum (respectively) of the matrix $\mathbf{X}$.

[20] As noted in European Central Bank (2012), around 60% of interbank repo transactions in the Euro area take place via CCP-based electronic trading.

[21] This iteration takes place in fictitious time as in standard tâtonnement processes. Banks do not trade during interest rate adjustment and trade only occurs once the equilibrium interest rate has been determined.

The problem of obtaining the interbank matrix $\mathbf{X}$ from its marginals (vectors $\mathbf{l}$ and $\mathbf{b}$) is not new in the literature dealing with empirical approaches to interbank contagion. The reason for this is that the bilateral exposure data in matrix $\mathbf{X}$ is not publicly available, whereas the overall lending to and borrowing from other banks is public information. A first approach to this problem constituted of using the maximum entropy method (see Upper (2011) for a summary). The latter operates under the logic that banks distribute their lending and borrowing as evenly as possible and therefore the number of connections in the network is maximized. As several studies working with real world interbank data have shown, such topology is not representative of actual interbank networks, which on the contrary tend to show a very low level of connectivity. More recent work has gone in the opposite direction of maximum entropy, trying instead to find the matrix that minimizes the density of the network (see for instance Anand et al. (2015)).

The problem can be cast as in terms of a system of linear equations of the form $\mathbf{Ax} = \mathbf{y}$, where $\mathbf{A}$ is an $2n \times N(N-1)$ matrix with $N-1$ ones per row and two ones per column (and zeros elsewhere), $\mathbf{x}$ is $N(N-1) \times 1$ vector containing all the non-diagonal elements of matrix $\mathbf{X}$, and $\mathbf{y}$ is a $2N \times 1$ vector which stacks the vectors $\mathbf{l}$ and $\mathbf{b}$. For $N > 3$ the conditions for the existence of a unique solution to the system will not be achieved, but at the same time, given the specific form of matrix $\mathbf{A}$, neither will the conditions for the *inexistence* of any solution. There will be therefore infinite solutions. This guarantees that by randomly pairing the banks one will eventually find a solution that delivers a matrix $\mathbf{X}$ consistent with the vectors $\mathbf{l}$ and $\mathbf{b}$. The question is then whether banks are actually paired randomly in real world interbank networks. There is fact no evidence for this; on the contrary, there are economic reasons implying non-randomness in the way banks relate to each other.

The matching algorithm we consider is the *closest matching*, or *minimum distance*, algorithm (henceforth CMA), which is in the spirit of recent approaches in that it minimizes the number of connections by construction, while at the same time having an underlying economic rationale behind the matching of specific banks. The rationale behind this mechanism lies in matching pairs of banks whose desired demand and supply are close in terms of size. The vectors of lending and borrowing are ordered in descending order and transactions are assigned. For the sake of argument, say banks i and j are the largest lender and borrower respectively, then the element (i, j) of the interbank matrix will be given by $x_{ij} = \min\{l_i, b_j\}$. This process goes over all pairs of banks and whatever residual desired amount that remains after a every transaction is stored for the next round of the algorithm.

Since in our setting, as in the real world, banks are on both sides of the market, some complications may arise. In particular, an issue which can emerge is that, because of the order in which the transactions are effected, a bank will eventually be "matched against itself" at the last stage of the algorithm. Of course this cannot be the case since, as mentioned earlier, we assume that banks do not trade with themselves. When we encounter such issue, the algorithm starts again from scratch but introduces a random swapping in the ordering of banks. From the argument developed in the paragraph above the achievement of solution is in this way guaranteed.

In this case matching takes place sequentially following the notion of deferred-acceptance established in Gale and Shapley (1962). The interbank trading matrix obtained by this method delivers a low level of connectivity, providing in fact a minimum density matrix. This low level of density or connectivity is in line with the one observed in the data. The CMA is also based on a stability rationale, as it is generally compatible with pair-wise efficiency and has been proposed in the seminal treaty of Shubik (1999) as most apt to capture clearing in borrowing and lending relations.[22]

3.3 Price Tâtonnement in the Market for Non-Liquid Assets

In this section we briefly describe the clearing process used for the non-liquid asset market, which is modelled along the lines of Cifuentes et al. (2005) and operates once a shock has hit the system. As mentioned earlier, the price of non-liquid assets is set to 1 when the financial system is set up. This is the price corresponding to zero aggregate sales and banks fulfilling regulatory requirements (i.e. the "status quo" price). The occurrence of shocks to banks' non-liquid asset holdings may force them to put some of their stock of assets on the market in order to fulfill regulatory requirements. This increases the supply of assets above demand. As a result the price adjusts to clear the market.

The logic of the mechanism can be described as follows. Consider the situation in which bank i is forced to sell non-liquid assets for an amount s_i in order to fulfill the equity requirement. An expression for s_i can be obtained by replacing n_i with $n_i - s_i$ in the denominator of Equation 3 and solving for s_i. From that it is straightforward to see that s_i will be decreasing in prices p, implying in turn that the aggregate sales function $S(p) = \sum_i s_i(p)$ is also decreasing in p. Defining the aggregate demand function as $\Theta(p) : [p, 1] \to [p, 1]$, an equilibrium price solves the following fixed point problem: $\Theta(p) = d^{-1}(S(p))$.

The price at which total aggregate sales are zero, namely $p = 1$, can certainly be considered one equilibrium price. But a key insight from Cifuentes et al. (2005) is that a second (stable) equilibrium price exists, to the extent that the supply curve $S(p)$ lies above the demand curve $D(p)$ for some range of values. The convergence to the second equilibrium price is guaranteed by using the following inverse demand function[23]:

$$p = exp(-\beta \sum_i s_i), \tag{16}$$

where β is a positive constant to scale the price responsiveness with respect to non-liquid assets

[22] In a previous version of this paper we also considered two alternative matching mechanisms, namely the maximum entropy algorithm and a random matching algorithm with a loading factor calibrated to obtain a density in between the extremes of CMA and maximum entropy. These two alternatives deliver networks with a significantly different topology. Results are available upon request.

[23] This function can be rationalized by assuming the existence of some noise traders in the market.

sold, and s_i is the amount of bank i's non-liquid assets sold on the market.

For an initial decline in prices to, say, p_0, banks will respond by putting an amount $S(p_0)$ on the market. But given Equation 16, this will in turn push the price down to $p_1 = d^{-1}(S(p_0))$. This generates further sales to the tune of $S(p_1)$. This process goes on until a new equilbrium price p^* is reached. For further details on the mechanism we refer the reader to the seminal contribution by Cifuentes et al. (2005).

3.4 Equilibrium Definition

Definition. A competitive equilibrium in our model is defined as follows:

(i) A quadruple (l_i, b_i, n_i, c_i) for each bank i that solves the optimization problem P.

(ii) A clearing price in the interbank market, r^l, which satisfies $B = L$, with $B = \sum_{i=1}^{N} b_i$ and $L = \sum_{i=1}^{N} l_i$.

(iii) A trading-matching algorithm for the interbank market.

(iv) A clearing price for the market of non-liquid assets, p, that solves the fixed point: $\Theta(p) = d^{-1}(s(p))$.

3.5 Risk Transmission Channels in the Model

Before proceeding with the simulation results, it is useful to highlight the main channels of risk transmission in this model. There are three channels which operate simultaneously; to fix ideas we start by describing the effects of real interlinkages.

First, a direct channel goes through the lending exposure in the interbank market. When bank i is hit by a shock which makes it unable to repay interbank debt, default losses are transmitted to all the banks exposed to i through interbank loans. Depending on the size of losses, these banks, in turn, might find themselves unable to fulfill their obligations in the interbank market.

The increase of default losses and in the uncertainty of debt repayment makes risk averse banks more cautious. They therefore hoard liquidity. The ensuing fall in the supply of liquidity increases the likelihood that banks will not honor their debts, reduces banks' resiliency to shocks and amplifies the cascading effects of losses. Notice that convex marginal objectives with respect to returns are also crucial in determining an increase in precautionary savings in the face of increasing uncertainty.

Liquidity shortage quickly turns into insolvency. Moreover, it reduces banks' exposure to non liquid assets. Eventually banks are forced to sell non-liquid assets if they do not meet regulatory requirements. If the sale of the assets is large enough, the market experiences a collapse of the asset price. This is the essence of pecuniary externalities, namely the fact that liquidity scarcity and the ensuing individual banks' decisions have an impact on market prices. In an environment in which banks' balance sheets are measured with mark-to-market accounting, the fall in the asset price induces accounting losses to all banks which have invested in the same asset. Accounting losses force other banks to sell non-liquid assets under distress. This vicious circle also contributes to turn a small shock into a spiralling chain of sales and losses. Three elements are crucial in determining the existence of fire sale externalities in our model. First, the presence of equity requirements affects market demand elasticities in a way that individual banks' decisions about asset sales do end up affecting market prices. Second, the tâtonnement process described above produces falls in asset prices whenever supply exceeds demand. Third, banks' balance sheet items are evaluated with a mark-to-market accounting procedure.

All the above-mentioned channels (credit interconnections among banks, liquidity hoarding and fire sales) have played an important role during the 2007 crisis. Caballero and Simsek (2013) for instance describe the origin of fire sale externalities in a model in which the complex financial architecture also induces uncertainty, which amplifies financial panic. Afonso and Shin (2011) instead focus on loss transmission due to direct exposure of banks in the money market and through liquidity hoarding. Our model merges those approaches and gains a full picture of the extent of the cascade following shocks to individual banks[24].

Notice that the mechanisms just described are in place even if the shock hits a single bank. However to produce a more realistic picture in the simulations presented below we assume a multinomial distribution of shocks to non-liquid assets: initial losses can therefore hit all banks and can also in principle be correlated. Therefore our numerical exercise will account for the quantitative relevance of contagion by assuming also asset risk commonality.

3.6 Systemic Risk

The 2007-8 crisis moved the attention of supervisory authorities from the too-big-to fail to the too-interconnected-to fail banks. In the past, systemically important banks were identified based on concentration indices such as the Herfindahl index. Nowadays systemically important banks are those who are highly interconnected with others. To measure the relevance of interconnections, an important distinction arises between ex ante and ex post metrics. Ex ante measures determine the contribution of each bank to systemic risk based on a time-t static configuration of the network. These measures are useful as they identify banks/nodes which can potentially be risk spreaders, but they have little predictive power, as they do not consider the transformations in the network topology following shocks. On the contrary ex post measures do so, hence they can be fruitfully used in stress tests. Overall ex ante measures can be used for preemptive actions, while ex post measures can be used to predict the possible extent of contagion in the aftermath of shocks, an information crucial to establish the correct implementation of post-crisis remedies.

Our focus here is on one ex post metric, namely the Shapley value[25]. In Appendix D we

[24] A short description of the shock transmission process is given in in Appendix C.

[25] See Shapley (1953) for the formal problem. Drehmann and Tarashev (2013) applied this concept to banking for the first time, and it was subsequently used by several authors.

report the performance in the numerical analysis of a set of ex ante metrics, namely network centrality measures, as well as their comparison with the Shapley value. The Shapley value comes from the literature on cooperative and non-cooperative game theory, and provides the contribution (through permutations) of each bank to an aggregate value. The latter in our case is the aggregate probability of default and is computed via the ratio of assets from all defaulting banks to total assets, $\Phi = \frac{\sum_{\Omega} assets_{\Omega}}{\sum_{i} assets_{i}}$, where $\Omega \in i$ identifies the set of defaulting banks (banks that cannot fulfill regulatory requirements even after selling all assets). One desirable property of the Shapley value is additivity, which in our case implies that the marginal contribution of each bank adds up to the aggregate default probability.

Formally the Shapley value is defined as follows. Define first C as a coalition of players which is a subset of the set definining all possible coalitions with N players (the latter denoted by C_N). In this spirit, C_{-i} stands for a coalition which does not include player/bank i. Next, define v^{Ψ} a function which maps subsets of players to the real numbers (i.e. $v^{\Psi}: 2^N \to \mathbb{R}$, where by convention it is assumed that $v(\emptyset) = 0$). This so called characteristic function will generate a value $v^{\Psi}(C)$ for every possible coalition C: in our case this value is systemic risk when the coalition C of banks is being shocked. Similarly, $v^{\Psi}(C_{-i})$ will indicate the value generated by a coalition which does not include bank i. With these elements in mind, the Shapley value for bank i can be expressed in the following way:

$$\Xi_i(v^{\Psi}) = \frac{1}{N!} \sum_{C \in C_N} \left(v^{\Psi}(C_{-i} \cup i) - v^{\Psi}(C_{-i}) \right) \tag{17}$$

where $v^{\Psi}(C_{-i} \cup i)$ is the value obtained by coalition C_{-i} but when also including bank i. That is, $\Xi_i(v^{\Psi})$ gives the average marginal contribution of player i over all possible coalitions of player set N. Note that the index Ψ denotes different possible shock scenarios, hence banks' contribution to systemic risk is computed conditional on a shock vector to the banking system.[26]

4 Baseline Scenario Results and Empirical Matching

In this section we present the baseline network configuration, which we characterize using synthetic metrics, namely density, average path length, assortativity, clustering, betweenness and eigenvector centrality. Additionally, we consider other items derived from the final configuration of the network which are useful in assessing its realism. In particular we consider the ratio of interbank assets to total assets, the equilibrium interest rate achieved through the interbank market tâtonnement

[26] As can be seen by the fact that the possible coalitions which can be formed with player set N is given by 2^N, the computation of the Shapley value is usually subject to the curse of dimensionality. For this reason it is normally approximated in numerical simulations by the average marginal contribution of players to the aggregate value over M randomly sampled permutations or coalitions, $\Xi_i(v^{\Psi}) \approx \hat{\Xi}_i(v^{\Psi}) = \frac{1}{M} \sum_{C \in C_M} \left(v^{\Psi}(C_i \cup i) - v^{\Psi}(C_{-i}) \right)$.

process, the number of intermediaries in the system (i.e. banks which both borrow and lend), and the subset of intermediaries which form the core of the system.[27]

Our primary goal is to verify that our banking network shares topological properties with the empirical counterparts. We indeed find that our model is able to replicate a number of stylized facts characterizing real world interbank networks (core-periphery structure, low density and dis-assortative behavior).

Before presenting the simulation results for the baseline structure, we describe the model calibration, which is largely based on banking and regulatory data. Table 1 summarizes calibrated values and shock distributions.

Following Drehmann and Tarashev (2013), the number of banks is set to 20. This keeps the system manageable in terms size (allowing us to track the behavior of different banks) and in terms of computation time. All policy related parameters are taken from the implementation of Basel III in Europe (see the *Regulation No 575/2013 of the European Parliament and of the Council* of 26 June 2013). The liquidity requirement (α), equity requirement ($gamma$), risk weights on non-liquid assets (ω_n) and interbank lending (ω_l) are set respectively to 10%, 8%, 0.2 and 1.[28]. We use data from Bureau van Dijk's Bankscope database to calibrate deposits and equity. We take the average of total assets for the period 2011-2013 for Euro Area banks, and use deposits and equity (again averaged over 2011-2013) of the top 20 banks in terms of assets.[29] The return on non-liquid assets is randomly drawn from a uniform distribution over the range $0 - 15\%$ (the variance is computed accordingly), whereas the vector of shocks to non-liquid assets, which is the starting point of the shock transmission process, is drawn from a multivariate normal distribution with a mean of 5, a variance of 25 and zero covariance (we draw 1000 shocks to evaluate the model). We set the loss given default parameter ξ to 0.5 (see for instance Memmel and Sachs (2013)), whereas for the expected probability of default and its variance we assign values of 0.5% and 0.3% respectively. Finally, the banks' risk aversion parameter σ is set equal to 2. For precautionary saving to arise such parameter must be larger than 1. Note also that the parameter β capturing the price responsiveness relative to non-liquid assets sold is endogenous and calculated as the number necessary to achieve a 10% drop if all non-liquid assets optimally chosen by banks are sold on the market (see Greenwood et al. (2015) and references therein for price responsiveness in fire sales processes).

[27] As noted by Craig and von Peter (2014), interbank markets typically present a tiered structure, and intermediation plays a key role in assessing that structure. In particular, an interbank market is tiered when there are banks which intermediate between other banks that are not directly connected. The two tiers thus formed are a core of densely connected banks, and a periphery of banks unconnected to each other directly but connected to the core. Core banks are therefore a strict subset of intermediaries: those intermediaries that serve to connect peripheral banks that would otherwise be disconnected from each other.

[28] The banks' capital buffer (on top of the equity requirement) is set to 1%

[29] The underlying data used to construct the averages is at the quarterly frequency, the highest frequency available for such data. The calibration is done with this frequency in mind. This has a bearing on the usefulness of systemic risk and systemic importance measures. For instance, at an extremely high frequency, systemic importance measures building directly on the matrix of interbank connections are likely to be very volatile and therefore lose most of their informational value.

Par./Var.	Description	Value
N	Number of banks in the system	20
α	Liquidity requirement ratio	0.10
ω_n	Risk weight on non-liquid assets	1
ω_l	Risk weight on interbank lending	0.20
γ	Equity requirement ratio	0.08
τ	Desired equity buffer	0.01
d_i	Bank deposits	Top20 EA
e_i	Bank equity	Top20 EA
σ	Bank risk aversion	2
ξ	Loss given default	0.5
$E[\delta]$	Expected default probability	0.005
σ_δ^2	Variance of default probability	0.003
r_i^n	Return on non-liquid assets	$U(0, 0.15)$
$\sigma_{r_i^b}^2$	Variance of r_i^n	$\frac{1}{12}(\max(r_i^n) - \min(r_i^n))^2$
Ψ	Shocks to non-liquid assets	$\aleph(\mathbf{5}, 25 * \mathbf{I})$

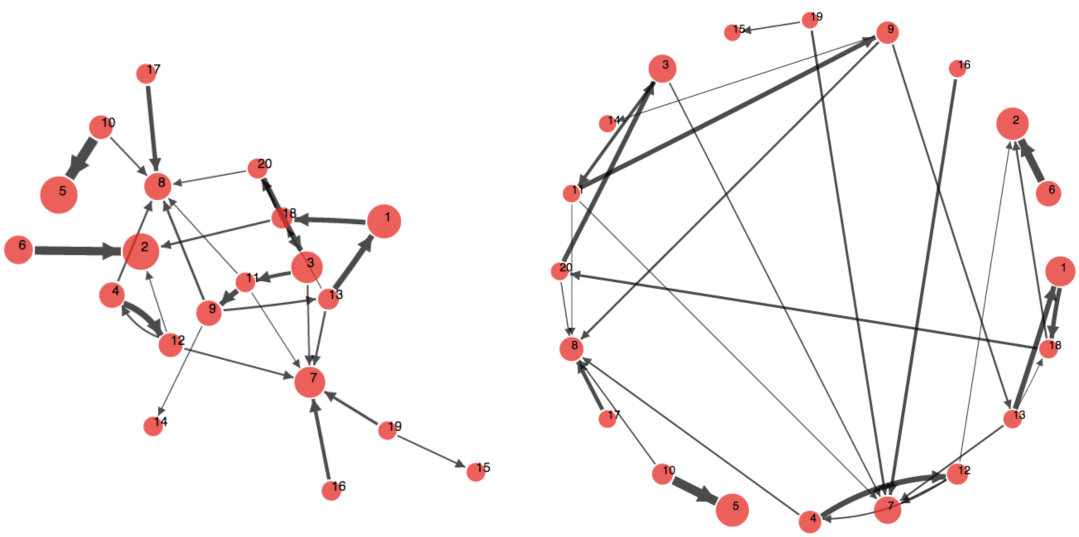
Table 1: Baseline calibration

We start by describing the partitions of banks into borrowers and lenders, the share of interbank assets over total assets and the equilibrium interbank rate (see also Table 2 below). Given the above calibration, the equilibrium interbank rate is 2.98%, in line with the pre-crisis average of EONIA. Interbank assets as a share of total assets stand at 23.7%, also in line with real world counterparts. There are 5 banks that only lend (banks $6, 10, 16, 17$ and 19), 6 that only borrow ($2, 5, 7, 8, 14$ and 15) and 9 intermediaries that both borrow and lend ($1, 3, 4, 9, 11, 12, 13, 18$ and 20). Generally speaking banks who borrow are those whose returns on non-liquid assets are high (and higher than returns on interbank lending). Since those have good investment opportunities they wish to invest and require liquidity beyond the one present in their portfolio. On the contrary banks decide to lend when the rate that they receive on bank lending is higher than the rate of return on non-liquid assets. The convexity of the optimization problem implies that internal solutions exist and banks can be on both sides of the market, namely being borrowers and lenders at the same time. Few large banks enter both sides of the market and act as central nodes: those banks have high returns on non-liquid assets, hence they wish to obtain liquidity for investment, but they also have large cash balances and are willing to lend to acquire a diversified portfolio.

4.1 Synthetic Measures of Network Architecture and Empirical Matching

Our next step is to describe the network topology by using synthetic network indicators.[30] Notice that synthetic metrics describing the network largely depend upon the banks' optimization problem and upon the matching algorithm. On the other hand, for the static network configuration the three contagion channels described previously do not play a role since they become operative only when banks are hit by shocks. The network response to shocks and the role of the contagion channels for systemic risk will be analysed in Section 5.

Figure 1 presents the baseline configuration with an interbank matrix computed via the closest matching algorithm, given the parameters from Table 1. Different nodes represent banks and their size is given by total assets. The width of arrows indicates the amounts transacted and an arrow going from i to j indicates that i is exposed to j through lending. The amount of links is not particularly high. In network parlance, the network exhibits low density: the density of the network is 7.37%, in line with the evidence from country-specific studies of interbank markets[31].



(a) Standard representation **(b)** Circle representation

Figure 1: Baseline network configuration

Table 2 shows results for the other synthetic metrics considered, given the baseline parameterization.

[30] To compute some of the network indicators we made use of the Brain Connectivity Toolbox and the MatlabBGL library.

[31] See for instance van Lelyveld and In't Veld (2012) for the Dutch case. Regardless of the specific number, a general finding from the literature is that interbank markets present low density.

Density (%)	7.37
Average Degree	1.40
Average Path Length	2.60
Betweenness Centrality (Av.)	7.10
Eigenvector Centrality (Av.)	0.13
Clustering Coefficient (Av.)	0.03
Assortativity	
out-in degree	-0.15
in-out degree	0.26
out-out degree	-0.31
in-in degree	-0.44
# Intermediaries	9
# Core Banks	3
Interbank Assets/Total Assets (%)	23.68
Equilibrium Interbank Rate (%)	2.98

Table 2: Network characteristics - Baseline setting

The first two network metrics are closely related. The density of the network captures the share of existing links over the total amount of possible links, whereas the average degree gives the average number of connections per bank. Both metrics proxy the extent of diversification in the network. By construction, the CMA network presents low density and hence a low average degree: a bank is connected on average to 1.4 other banks.

The average path length is the mean shortest path between pairs of nodes. It gives an idea of the ease with which one can expect to get from a given node to any other given node. In our case this number is 2.6, implying that the average bank is almost 3 connections away. The average path length is small, in line with real-world interbank networks (see Alves et al. (2013) or Boss et al. (2004) among others). This implies that exposure is not far away for the average bank in the network.

Betweenness and eigenvector centrality are computed as averages for all nodes in the network. The CMA network features high betweenness and eigenvector centrality since a few banks act as gatekeepers.

The clustering coefficient measures the tendency of neighbors of a given node to connect to each other, thereby generating a cluster of connections. For our network configuration the average clustering coefficient is low, especially in relation to other types of networks (for instance, trade networks), and in line with evidence on real-world interbank networks.

The assortativity coefficient aims at capturing the tendency of high-degree nodes to be linked to other high-degree nodes. As noted by Bargigli et al. (2015), interbank networks tend to be dis-assortative, implying that high-degree nodes tend to connect to other high-degree nodes less frequently than would be expected under the assumption of a random rewiring of the network that preserves the nodes' degrees. With the exception of the *in-out* coefficient, which presents positive assortativity, our network presents in fact dis-assortative behavior. These results are in line with those observed in the data (see for instance Bargigli et al. (2015) or Alves et al. (2013) among others). Notice that dis-assortative behavior is associated with core-periphery structures; this is true both in the data and in our model. As already mentioned above, a necessary condition for the presence of a core-periphery structure is to have banks which both borrow and lend, i.e. to have intermediaries. Out of the 20 banks in our model, 9 are intermediaries. Furthermore, from these 9 banks, 3 constitute the core of the network.[32]

To sum up our network shares most synthetic indicators with the empirical counterparts. Notably the network is characterized by low density, low clustering, low average path length, dis-assortative behavior and a core-periphery structure in which the core is a strict subset of all intermediaries. Further results for the simulation of the baseline network can be found in Appendix D.

5 Model Response to Shocks

An essential prerequisite of prudential regulation consists in measuring systemic risk and identifying systemically important banks. Assessing the contribution of each bank to risk propagation is indeed a crucial aspect of the inspecting activity that supervisors conduct to prevent crises. To this aim and prior to the analysis of the prudential policy we present some metrics that measure the contribution of each bank to systemic risk or that allow the supervisor to detect systemically important intermediaries. In this section we focus specifically on the Shapley value. Given the system-wide default probability following a multinomial distribution of banks' shocks, the Shapley value determines the contribution of each bank to it.[33]

Figure 2 presents each bank's contribution to systemic risk, based on the Shapley value methodology. The number of permutations considered for the computation of the Shapley Value was set to 1000. The clearing algorithm for the interbank market used is that of Eisenberg and Noe (2001). We simulate shocks to the value of non-liquid assets with multinomial distributions. In response to those shocks all channels of contagion are activated. First and foremost, banks become more cautious and start to hoard liquidity thereby producing a credit crunch in the interbank market. The fall in liquidity supply together with the adverse shocks on some banks' assets produces many adverse effects: some banks stop honoring their debt obligations, most banks de-leverage, and some banks sell their non-liquid assets to meet equity and liquidity requirements. All those actions trig-

[32] Our conception of the core follows that of the seminal work of Craig and von Peter (2014). We thank Ben Craig for sharing the code for the computation of the core-periphery structure.

[33] Other indicators can be used to identify systemically important banks. In Appendix D we present results for one such type of indicators, namely network centrality metrics, and compare the results with those obtained from the Shapley value analysis.

ger further losses. The liquidity hoarding reduces the system's resiliency to shocks: banks who do not repay their debt transmit direct losses to exposed lenders; fire sales of non-liquid assets, by triggering falls in assets prices, transmit indirect losses to the balance sheets of other banks.

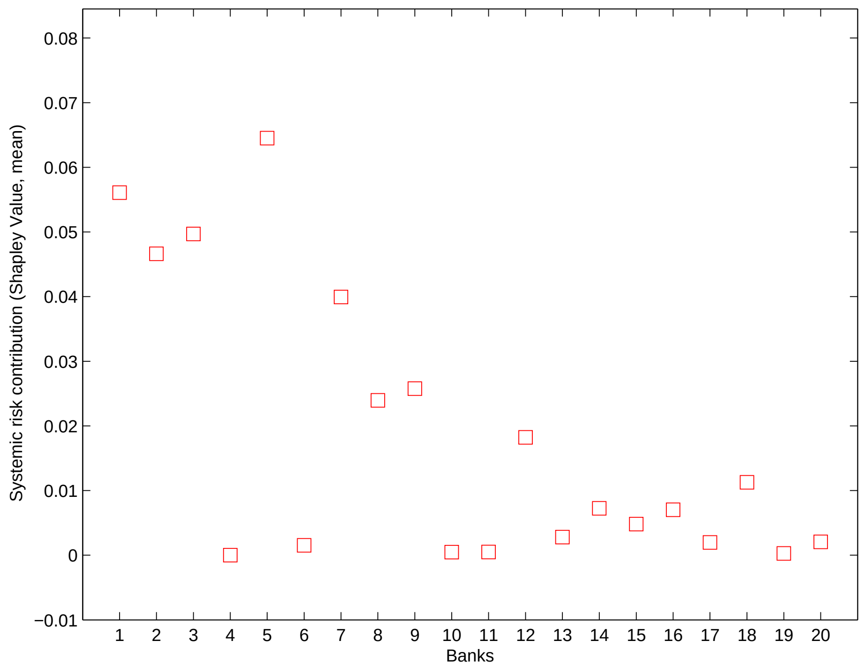


Figure 2: Contribution to systemic risk (mean Shapley Value) by bank

By jointly analyzing the data in Figure 2 and the banks' optimal portfolio allocations as reported in Table 4 in Appendix D we find that the banks which contribute the most to systemic risk are the ones which both borrow in the interbank market and invest highly in non-liquid assets.[34] Generally speaking we find a strong connection between Shapley value and total assets. Interbank borrowing increases the extent of risk transmission through direct interconnections, while investment in non-liquid assets increases the extent of risk transmission via fire sale externalities. The more banks borrow and the more banks invest in non-liquid assets, the larger is their contribution to cascading defaults and to systemic risk. The rationale behind this is as follows. Banks which leverage more in the interbank market are clearly more exposed to the risk of default on interbank debts. The larger is the size of debt default the larger are the losses that banks transmit to their counterparts. Borrowing banks therefore contribute to systemic risk since they are the vehicle of network/interconnection externalities. On the other hand, banks which invest more in non-liquid assets transmit risks since they are the vehicle of pecuniary externalities. The higher is the fraction of non-liquid asset investment, the higher is the negative impact that banks' fire sales have on market prices. The

[34] Usually those are also the banks with the higher returns on non-liquid assets investment.

higher is the collapse in market prices, the higher are the accounting losses experienced by all other banks due to asset commonality and mark-to-market accounting. Notice that banks which invest and borrow much are also those with the highest returns on non-liquid assets investment. As banks invest more they also grow in size, consequently there is also a connection between banks' size and systemic risk. Figure 2 (observed in combination with total assets as from Table 4 which presents the optimal balance sheet structure in the baseline setting) shows for instance that smaller banks tend to contribute less to systemic risk. While the Shapley value shows a strong connection to total assets, the connection to other balance sheet items or relevant balance sheet ratios is not particularly strong (see Figure 4 in Appendix D.2).[35] To assess the role of banks' risk aversion and precautionary savings on the transmission of risk we present the main results for systemic risk by comparing the models with and without risk averse banks: see Appendix E. Generally speaking systemic risk is higher with risk averse banks. In the face of uncertainty banks' marginal utility from hoarding liquidity increases. The fall in interbank supply drives interbank rates up, which in turn increases debt default rates. Introducing convex preferences generally increases the degree of non-linearity featured by the model.

To test the robustness of the Shapley value we compute the ranking of systemically important banks also using alternative metrics, namely network centrality indicators. Due to space considerations, simulation results for those are presented and discussed in Appendix D.

6 Policy Analysis: Stability versus Efficiency

Recent guidelines on prudential regulation from Basel III include requirement ratios both for equity and for liquidity. A crucial policy question is whether changing the regulatory requirements affects systemic risk and the contribution of each bank to it. In setting the level of the regulatory requirements there are clearly trade-offs. For instance, higher equity requirements might be beneficial since they reduce the extent of banks' leverage (thereby reducing direct interconnections) and increase the share of assets potentially able to absorb losses. On the other hand, higher equity requirements imply that banks can invest less and that in the face of shocks the extent of fire sale increases with respect to the tightness of the regulatory constraint. Similar trade-offs apply to liquidity requirements.[36]

We inspect the variations in systemic risk and in the optimal allocation for different values of the liquidity requirement α and of the equity requirement γ. As in the baseline setting, the number of permutations for the computation of the Shapley Value is set to 1000.

[35]This holds irrespective of the matching algorithm used: in exercises not reported here we have computed the interbank matrix using other matching algorithms (which deliver a different network topology) and the qualitative message stays unaltered.

[36]Note that in our model the investment in external non-liquid assets is a proxy for the connection of the banking system with the real economy. We thereby take this as a measure of efficiency and as a crude substitute for welfare.

Table 3 summarizes the main results from the policy experiments. To the left (right) we have the results from changes in the liquidity (equity) requirement. The different panels (rows) represent, respectively: total systemic risk, interbank assets as a share of total assets, non-liquid assets as a share of equity, the equilibrium interest rate, leverage and network density.

We start by examining how overall systemic risk and the contribution of each bank to it change when altering the two policy parameters. At first glance, overall systemic risk shows a downward trend when we increase the liquidity parameter α. That said, as is obvious from the charts, starting from values around 0.2 systemic risk exhibits a jig-saw behaviour within this general downward trend. Such behaviour is not present in the linear model with risk neutrality and we therefore attribute it to the non-linearities embedded in the set-up of our model. There are some banks that always contribute to systemic risk (mostly banks 1, 2, 3, 5, 12 and 16, see Figure 7). The rationale for the results is as follows. As banks must hold more liquidity for precautionary motives, their exposure in the interbank market declines, though this is not reflected in interbank assets as a share of total assets since the reduction in non-liquid assets is quite substantial (see the upper right panel in Table 3). The interbank interest rate increases due to the scarce supply of liquidity (see lower left panel in Table 3) and banks' investment in non-liquid assets declines as available liquidity falls. Overall, there is a strong reduction in the scope for fire sale externalities and a relatively milder increase in the scope for network externalities. The ratio of non-liquid assets to equity is halved for the range of values of α under consideration, pointing to the trade off between stability (as proxied by systemic risk) and efficiency (as proxied by aggregate investment in non-liquid assets).

Results are somehow more complex when we increase the equity requirement, γ. As this parameter increases, overall systemic risk declines over an initial range, but it stays flat after roughly 0.13. Banks leverage less and the interbank interest rate declines as the demand of liquid funds has declined. This reduces the overall scope for transmitting default losses, and in fact interbank lending as a percentage of assets reaches very low values (see middle figure in the second row in Table 3). However, banks also reduce the amount of liquid assets (not shown here), while keeping the amount of non-liquid asset investment roughly unchanged in terms of equity for an initial range and then only reducing it slightly (see Table 3 and compare the y-axis of the two upper right panels).[37] The scope of risk transmission through fire sales is therefore only slightly reduced. Increasing the equity requirement above 10% seems to have a non-negligible impact on systemic risk, while at the same time not reducing efficiency as strongly as with increases in the liquidity requirement. As for the contribution of each bank to overall systemic risk (see Shapley values in Figure 8) we observe that, while most banks tend to transmit less risk as γ increases, others instead tend to contribute more.

[37]Notice that in our model raising equities does not entail adjustment costs. In reality and depending on the degree of financial market development some adjustment costs might render equity adjustment stickier. If so, it is possible that in face of increases in equity requirements banks might decide to partly increase equities and partly reduce their asset portfolio in order to rebalance the ratio. In any case we would observe a stronger fall in non-liquid assets under an increase in equity requirements than under an increase in liquidity requirements.

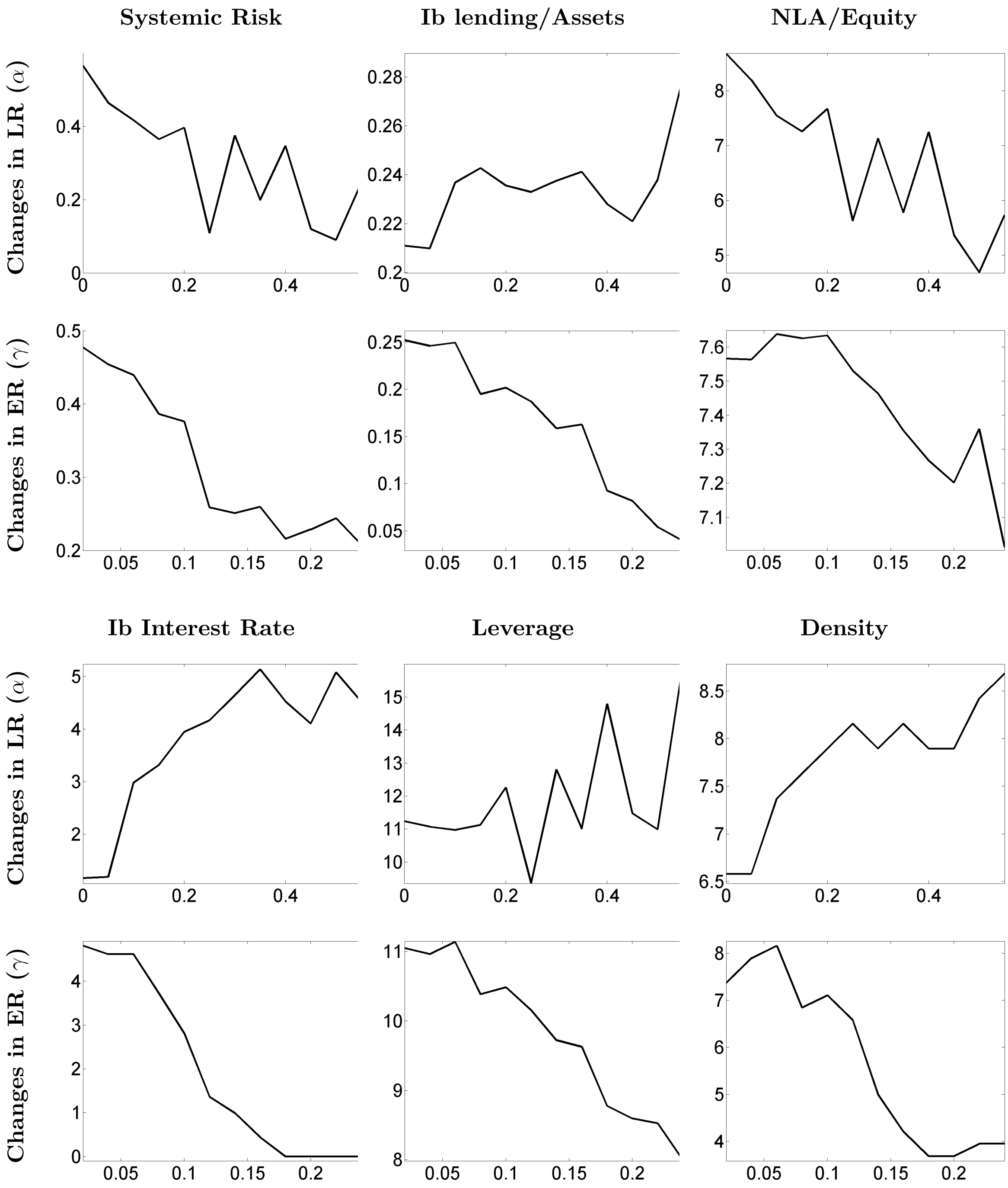


Table 3: Main results from policy analysis. Changes in the liquidity requirement (LR) and equity requirement (ER) are presented in the x-axis.

Since all banks are less exposed to the interbank market the scope of loss cascades through network linkages is reduced. On the other hand some banks invest more in non-liquid assets. This exposes the latter to the swings in the market price for non-liquid assets and increases the probability that they will engage in fire sales.

The lower right panels of Table 3 present the evolution of network density for the two policy experiments we entertain[38]. For changes in the liquidity requirement, network density presents an increasing trend, whereas for changes in the equity requirement there is an almost constant reduction in density, which is roughly halved over the range of values considered. While the upper limit for network density is roughly the same for the two policy exercises, it is worth noting that in the case of changes in the liquidity requirement, density never falls below the starting value of approximately 6.5%, whereas it falls to almost 3.5% when increasing the equity requirement. When changing the equity requirement there is a noticeable drop starting at around $\gamma = 0.12$. The reason for this can be seen in Figure 6b in Appendix F. The number of active banks in the interbank market drops substantially, in particular those banks that both borrow and lend. If we take the number of banks on both sides of the market as a proxy for intermediation activity, Figure 6b shows that intermediation reaches a peak when $\gamma = 0.12$. As the equity requirement increases less banks are active in the market and the ones that are actually active demand less liquidity relative to existing supply, forcing the continuous downward trend in the interbank rate that we see in Table 3.

As Figure 6a shows, no such development occurs when increasing the liquidity requirement. This essentially leaves the number of active banks unchanged. When the liquidity requirement increases there seem to be two countervailing forces that balance each other. As the liquidity requirement raises, banks supply less liquidity in the interbank market and this has a depressing effect on density and other measures such as closeness (not shown here). On the other hand, some banks increase their demand of liquid funds driving the interbank rate up and inducing other banks to substitute investment in non-liquid assets with interbank lending. This asset substitution effect increases the available liquidity in the interbank market (as shown in Table 3), which in turn has a positive impact on density and related measures.

To sum up, increasing the liquidity requirement unambiguously reduces systemic risk as it notably reduces the investment in non-liquid assets while only marginally increasing the scope for network externalities. The fall in the overall non-liquid asset investment shows however that an increase in the liquidity requirement reduces system efficiency. An increase in the equity requirement also decreases systemic risk (though the latter remains flat after $\gamma = 0.13$), but without a substantial decrease in efficiency.

[38] Average degree, path length and clustering coefficients paint a very similar picture so we left them out for the sake of space.

6.1 Systemic Risk and Contagion Channels

To assess the contribution of each of the channels considered (liquidity hoarding, interconnections and fire sales) we compare the evolution of systemic risk (under different values for α and γ) under four alternative models (see Appendix E). *Model 1* is the benchmark considered so far. *Model 2* considers risk neutral banks with a linear objective function, thereby eliminating the liquidity hoarding channel and eliminating the possibility that banks act on both sides of the market. *Model 3* eliminates investment in non-liquid assets in order to shut off the fire sale channel. Finally, *Model 4* is a small variation on *Model 3* in which the risk aversion parameter σ is set to zero. We can summarize the difference in results as follows. First, the benchmark model (with all contagion channels) shows larger swings in the changes of systemic risk with respect to α and γ. This is due to the fact that the presence of risk averse agents by triggering precautionary saving features higher non-linearities. Second, in *Model 4* systemic risk increases with respect to increases in α. This is empirically puzzling, although it is internally consistent with the assumptions of model 4, namely the absence of other investment opportunities beyond those in non-liquid assets and the assumption of $\sigma = 0$. As the liquidity requirement increases, banks which are short of funds increase their demand of interbank borrowing. This raises the interbank rate and makes interbank lending attractive for banks which have excess liquidity. Overall network linkages in the interbank market increase and so does contagion of default risk.

To summarize our benchmark model has two important appealing features. First, it generates realistic amplifications of risk and features non-linearity in transmission channels: both are realistic features of banking panics triggered by contagion channels. Second, and contrary to alternative models considered, it provides reasonable predictions for the response of the network to changes in policy regulations.

7 Concluding Remarks

We have analyzed a banking network model featuring risk transmission via different channels. Banks in our model are risk averse and solve a concave optimal portfolio problem. The individual optimization problems and the market clearing processes deliver a matrix of network links in the interbank market. Each bank can be both borrower and lender vis-à-vis different counterparties. Shocks to one bank are transmitted through defaults on interbank debt, through price collapses of non-liquid assets triggered by fire sales or through liquidity hoarding. Clearing in the market takes place through a price tâtonnement iterative process and through a trading matching algorithm, namely closest matching (or minimum distance). The network thus obtained resembles some characteristics from the empirical counterparts. In particular, it presents low density, low average degree, dis-assortative behaviour and a core-periphery structure.

We use our banking network to assess the role of prudential regulations in reducing systemic risk. We find that increasing the liquidity requirement unequivocally reduces systemic risk and the contribution of each bank to it. As banks must hold more liquidity for precautionary motives, their exposure in the interbank market declines, though this is not reflected in interbank assets as a share of total assets as the reduction in non-liquid assets is quite substantial. The former limits somewhat the scope for network externalities, whereas the latter substantially reduces the scope for pecuniary externalities. The reduction in non-liquid assets is so strong that there is an associated cost to it in terms of efficiency of the system, highlighting the existing trade-off between stability and efficiency. An increase in the equity requirement instead does not present this strong trade-off. Systemic risk decreases, in particular for an initial range of values of γ. The scope for network externalities is persistently reduced as the share of interbank assets over total assets steadily declines to reach very low values in the upper range of γ. While there is also a slight reduction in the scope for fire sales externalities, the reduction in non-liquid assets is relatively minor. The system becomes more homogenous and the potential damage from interbank market collapses is markedly reduced. This comes at the expense of having less banks trade in the interbank market, with an associated reduction in its density.

We have explored the effects of contagion and risk transmission stemming from the asset side of banks' balance sheets. Incorporating risk originating from the liability side would take our model one step further in the direction of realism. We leave this avenue for future research.

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A Deviation from the Modigliani-Miller theorem

Like firms, banks have a capital structure and it is legitimate to ask whether their value and risk profile is independent from their capital structure – i.e. whether the Modigliani-Miller theorem holds true for banks in our framework. In fact it is rather easy to prove that the theorem is violated in our case. The main reason for the departure stems from the bankruptcy costs associated with interbank debt. In our framework, banks can fund themselves through deposits and through interbank borrowing. The returns on deposits are fixed and constant: for generality we can set it equal to r^d. On the other side the cost of interbank borrowing for a single bank contains a premium for bankruptcy and is equal to $r^l + r^p = \frac{1}{1-\xi\delta} r^l$ (for simplicity we skip bank subindices). Notice that, while the bankruptcy premium r^p is derived so that the expected value of interbank lending is equalized to the return of a safe asset, the risk profile of interbank borrowing is in fact very different from that of other funding means, in particular deposits which are risk free in our set-up.

To prove that the capital structure matters for the banks' risk profile we proceed as follows. We define Q as the expected return on banks' asset reflecting also banks' risk and we define as V the market value of banks' securities. By construction the market value of banks' securities must equalize the sum of deposits and interbank borrowing: $V = d + b$. The Modigliani-Miller theorem states that the value of the bank shall be independent from the capital structure. To see if this is the case let's take two extreme cases. Suppose one bank funds itself entirely through deposits and a second bank funds itself entirely through interbank borrowing. We start by assuming that the two banks have the same market value and will instead show that this brings us to a contradiction. Define V_1 as the market value of bank 1 and V_2 as the market value of bank 2 and start by assuming that $V_1 = V_2$. For bank 1 it must be true that:

$$V_1 = d = \frac{Q}{r^d}$$

For bank 2 it must be true that:

$$V_2 = b = \frac{Q}{r^l + r^p}$$

Since the return of interbank borrowing features a premium over the lending rate it follows by construction that the two banks cannot have the same market value and the same risk profile.

B Model's Visual Representation

C Shock Transmission

The shock tranmission process can be succintly summarized as follows. After the vector of shocks is drawn the supply of non-liquid assets will be affected and therefore the price will have to be adjusted.

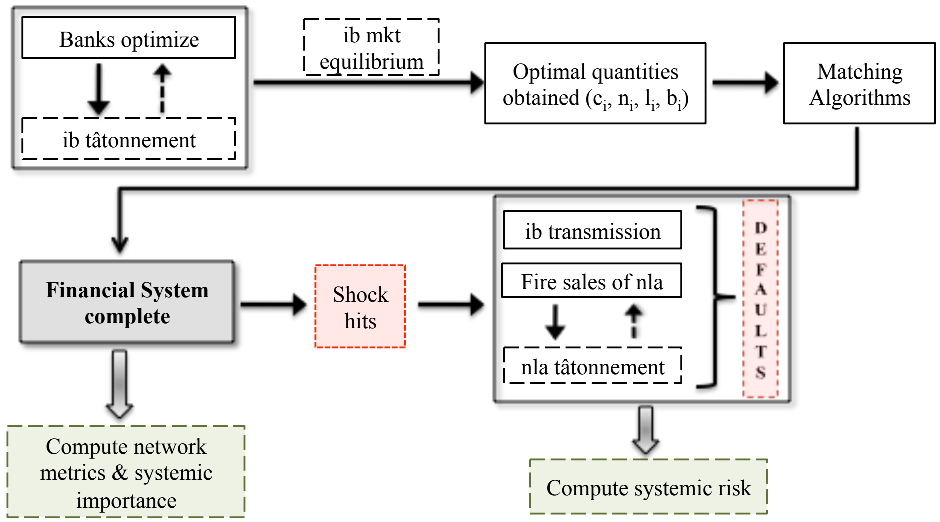


Figure 3: A bird's eye view of the model.

Following such adjustment, some banks may not be able to fulfill their interbank commitments. Such banks will liquidate their entire non-liquid asset holdings, pay as much as they can to interbank creditors and be added to the default set. The interbank adjustment is done following the now classic algorithm outlined in Eisenberg and Noe (2001). Note that, at this stage, interbank connections are taken as given and banks are not re-optimizing; changes to the interbank market structure are at this point the result of applying the clearing mechanism of Eisenberg and Noe (2001). At the same time, many banks may not be able to fulfill the equity requirement. Within this group, two sub-groups may be distinguished. First there are those banks that after selling part of their non-liquid asset holdings will be able to fulfill the equity requirement; the second group cannot fulfill the requirement even after selling all their non-liquid assets. The former group will just liquidate what it needs in order to comply with requirements, whereas the latter group will liquidate all and be added to the default set. All the non-liquid assets put on the market by all banks will be used for a recalculation of the price p and start a new round of the transmission process. When no more defaults occur the algorithm stops and systemic risk is computed as set out in the main text.

D Additional results for baseline scenario

D.1 Balance sheet characteristics and systemic importance ranking

	Banks																			
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Cash	56.9	53.6	48.7	30.5	57.0	46.6	36.4	20.5	22.9	25.8	3.9	14.3	12.7	10.7	6.8	12.1	12.9	6.7	7.3	7.7
Non-liquid assets	606.0	807.3	529.2	211.9	838.3	296.6	250.8	428.7	284.9	40.7	8.2	252.8	24.0	111.1	88.9	51.8	63.1	111.1	100.0	11.6
Interbank lend.	92.3	0.0	74.8	144.7	0.0	175.8	153.6	0.0	64.6	234.5	46.9	44.7	138.0	0.0	0.0	62.1	59.0	0.0	0.0	66.7
Interbank borr.	130.7	234.9	117.2	29.1	244.3	0.0	19.8	196.2	117.5	0.0	0.0	145.8	31.7	4.8	19.7	0.0	0.0	40.8	25.3	0.0
Total Assets (A)	755.2	860.9	652.7	387.1	895.3	519.0	440.8	449.2	372.5	301.0	59.0	311.8	174.7	121.8	95.7	126.0	135.0	117.8	107.3	86.0
Equity	55.6	90.0	48.5	53.0	81.0	53.0	57.0	48.0	26.0	43.0	20.0	23.0	16.0	10.0	8.0	5.0	6.0	10.0	9.0	9.0
Leverage	13.6	9.6	13.5	7.3	11.1	9.8	7.7	9.4	14.3	7.0	3.0	13.6	10.9	12.2	12.0	25.2	22.5	11.8	11.9	9.6
Liquid assets/A (%)	19.8	6.2	18.9	45.3	6.4	42.9	43.1	4.6	23.5	86.5	86.2	18.9	86.3	8.8	7.1	58.9	53.3	5.7	6.8	86.5
Interbank lend./A (%)	12.2	0.0	11.5	37.4	0.0	33.9	34.8	0.0	17.4	77.9	79.6	14.3	79.0	0.0	0.0	49.3	43.7	0.0	0.0	77.6
Interb. borr./Liab. (%)	18.7	30.5	19.4	8.7	30.0	0.0	5.1	48.9	33.9	0.0	0.0	50.5	20.0	4.3	22.5	0.0	0.0	37.9	25.7	0.0
Nla/Dep. (%)	106.5	150.6	108.7	69.5	147.1	63.6	68.9	209.1	124.4	15.8	20.9	176.8	18.9	103.8	130.7	42.8	48.9	165.8	137.0	15.0
Nla/Equity (%)	1090.7	897.0	1092.1	399.9	1035.0	559.6	440.0	893.1	1095.9	94.7	40.8	1099.3	149.9	1111.1	1111.1	1035.2	1051.0	1111.1	1111.1	128.8
Interb. lend./Equity (%)	166.1	0.0	154.3	273.0	0.0	331.7	269.4	0.0	248.6	545.3	234.7	194.3	862.8	0.0	0.0	1242.8	984.0	0.0	0.0	741.2

Table 4: Optimal balance sheet items - Baseline setting

Centrality	**Banks**																			
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
In-degree	5	3	6	7	8	16	1	2	9	17	10	11	12	13	14	18	19	4	20	15
Out-degree	11	15	5	6	16	12	17	18	1	7	2	3	4	19	20	13	14	8	9	10
In-out degree	12	8	9	10	15	16	1	2	3	13	4	5	6	17	18	19	20	7	14	11
Closeness-in	13	4	11	9	15	16	2	3	12	17	10	14	8	5	7	18	19	1	20	6
Closeness-out	11	15	6	10	16	14	17	18	3	9	2	4	1	19	20	12	13	7	8	5
Closeness-in-out	14	8	11	12	20	19	4	5	6	15	3	7	1	9	16	17	18	2	13	10
Betweenness	9	10	3	8	11	12	13	14	2	15	4	7	6	16	17	18	19	1	20	5
Eigenvector (left)	5	9	3	12	13	14	10	6	1	15	4	16	8	11	17	18	19	2	20	7
Eigenvector (right)	3	8	5	9	10	11	12	13	7	14	4	15	1	16	17	18	19	6	20	2

Table 5: Systemic importance ranking by network centrality measures - Baseline setting

D.2 Additional results on Shapley value and systemic importance

Figure 4 plots the Shapley value versus bank characteristics. Results point to a strong connection with total assets as discussed in the main body of the paper. The connection to other balance sheet items is rather weak.

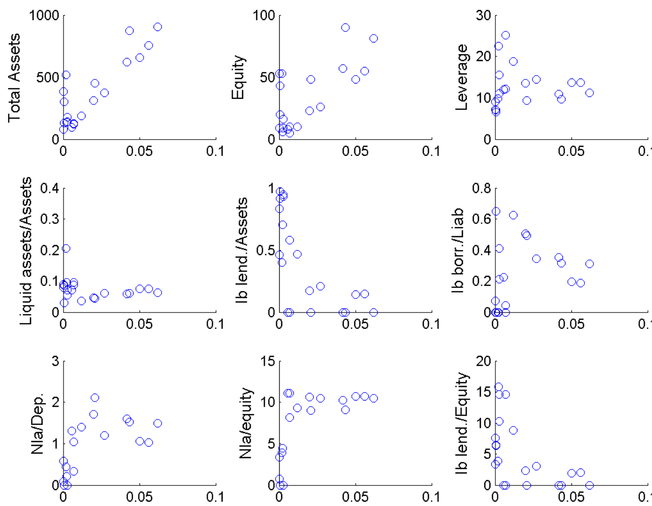


Figure 4: SV vs. bank characteristics

For systemic importance measures we consider network centrality indicators. In graph theory and network analysis the centrality of a vertex or node measures its relative importance within the graph. In particular, we consider the following measures: degree, closeness, betweenness and eigenvector centrality.[39] Degree centrality captures the number of connections that a bank has. In networks in which the direction of links matter, like ours, it can be divided into in- and out-degree. The former accounts for the number of links "arriving" to a node, whereas the latter quantifies the number of links "leaving" a node. Closeness centrality assesses the importance of nodes based on how reachable they are from all other nodes (i.e. how "close" they are). Betweenness centrality gauges the relative importance of nodes based on how often they lie in paths connecting other nodes (i.e. how important they are as "gatekeepers"). Finally, eigenvector centrality is a generalization of degree centrality which captures the idea that connections to other nodes which are themselves well connected should carry more weight.[40]

Table 5 above presents the ranking of systemic importance for the baseline setting and for all

[39] With this choice we cover the range of possible measures based on standard taxonomy (see for instance Alves et al. (2013)).

[40] In directed networks one can also subdivide closeness and eigenvector centrality, the former into *in* and *out* versions, the latter into *left* and *right* eigenvectors.

the measures considered. Depending on the measure one chooses to focus on, the assessment differs substantially for many banks. At one extreme we have for instance bank 7, which can be ranked first according to one measure, and up to seventeenth by another. There are some banks that are consistently ranked high or low (see for instance bank 18 for the former and bank 17 for the latter).

Another interesting question is whether systemic importance measures (i.e. centrality indicators) and systemic risk measures (i.e. Shapley value) deliver a consistent ranking. Figure 5 sheds light on this issue by plotting the Shapley value versus the different network centrality measures considered.[41] The bottom line is that there is no apparent connection between the ranking provided by the two types of measures. While this may seem disappointing at first glance, one should bear in mind that these measures are not only different algebraically, but also conceptually. Systemic importance measures are of an ex-ante nature in the sense that all that is needed for their computation is a matrix representing the connections between banks. Importantly, to construct these measures there is no need for a shock to hit the system and thereby no need either for the specification of behavioral responses. They are in this sense also static. For systemic risk indicators to be computed one needs indeed to measure risk, and to that end assume some kind of shock to the system.[42] Furthermore, behavioral responses of some sort are needed for the shock process to converge. In this respect this type of measures have a more dynamic flavor.

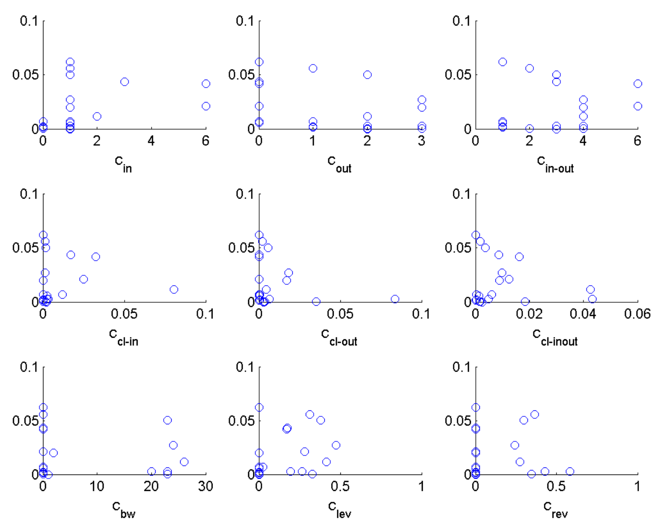


Figure 5: SV vs. centrality measures

[41]In the working paper version of this paper we also perform the comparison with other family of systemic importance indicators, namely input-output-based measures, and the message remains unaltered.

[42]This can be for example the targeted exogenous failure of a given institution, the sequential exogenous failure of all institutions, or as we explore in this paper, multinomial shocks to all banks simultaneously.

E Model Comparison

In this section we compare the results from different models to illustrate some differences. We perform a policy analysis in the same fashion as in the main body of the paper. For all models considered the interbank matrix was obtained by means of the CMA algorithm, and the shock simulation involves 1000 realizations of the shock vector. We consider the following four alternative models:

- **Model 1**: this model is the one presented in the main body of the paper, featuring risk averse banks and the interaction of fire sales and network externalities.
- **Model 2**: this model has risk neutral instead of risk averse banks, hence the objective function is linear and simply given by utility of expected profits, which in this case is equal to expected utility of profits. The constraints remain the same, and fire sales and interbank contagion are also kept. It is worth noting that in this model there are no banks that participate on both sides of the market simultaneously, i.e. they are either borrowers or lenders.
- **Model 3**: this model is similar to *Model 1* but it eliminates the fire sales channel. Non-liquid assets are no longer a choice variable of banks and are instead calibrated by the values banks would have chosen if given the chance. Once a shock hits banks cannot sell the assets and the transmission of distress takes place only through the interbank channel.
- **Model 4**: this model is a small variation of *Model 3*. In particular, we set the risk aversion parameter to $\sigma = 0$.

Results from the comparison exercise are summarized in Table 6, which presents the effects of changes in the liquidity and equity requirements on systemic risk, interbank lending over total assets and non-liquid assets over equity.

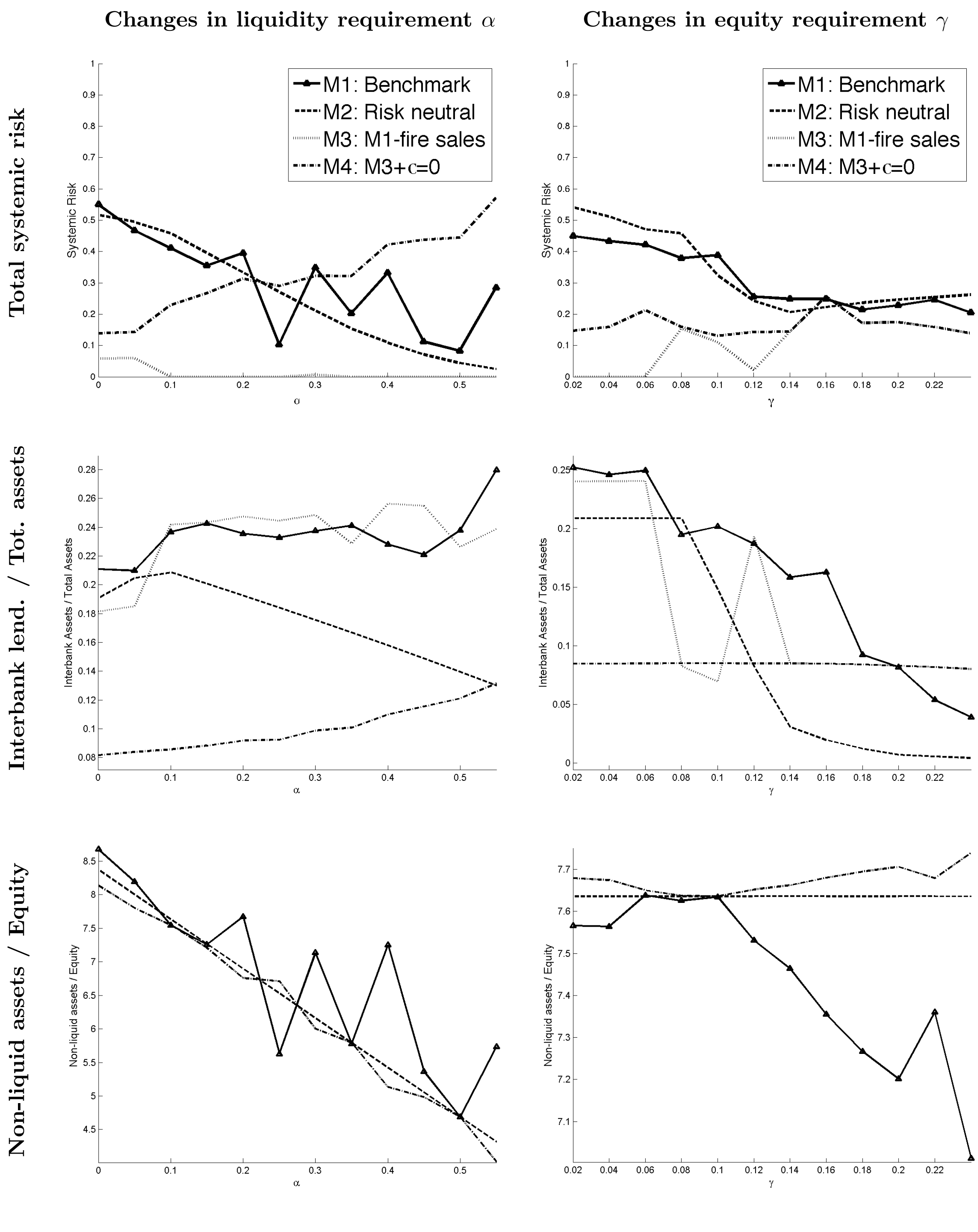
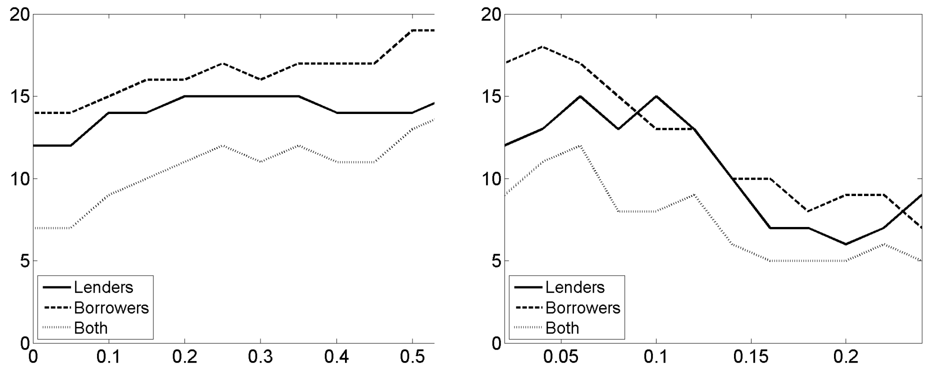


Table 6: Model Comparison

F Additional results for comparative static analysis



(a) Changes in LR (α)   **(b)** Changes in ER (γ)

Figure 6: Number of active banks in interbank market for different values of α and γ

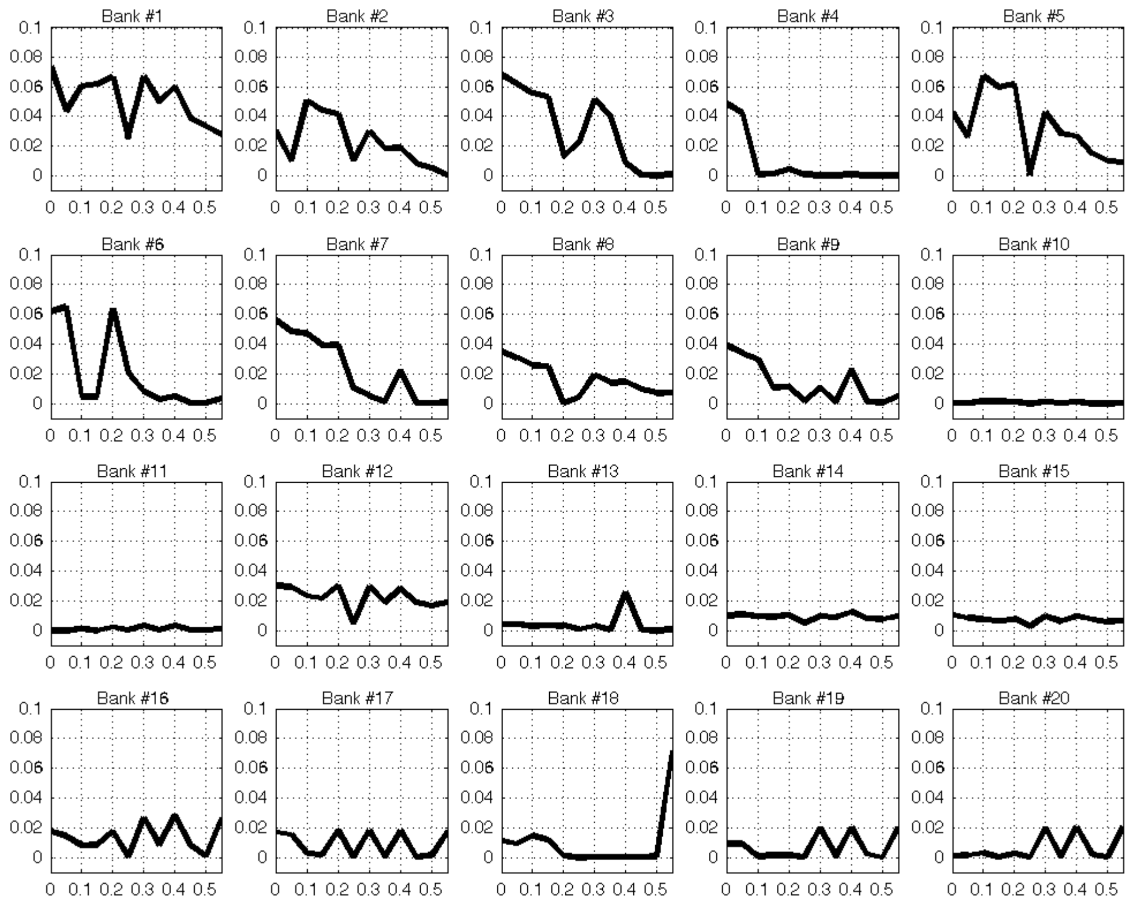


Figure 7: Contribution to systemic risk (Shapley Value, y axis) by bank for different values of α (x axis)

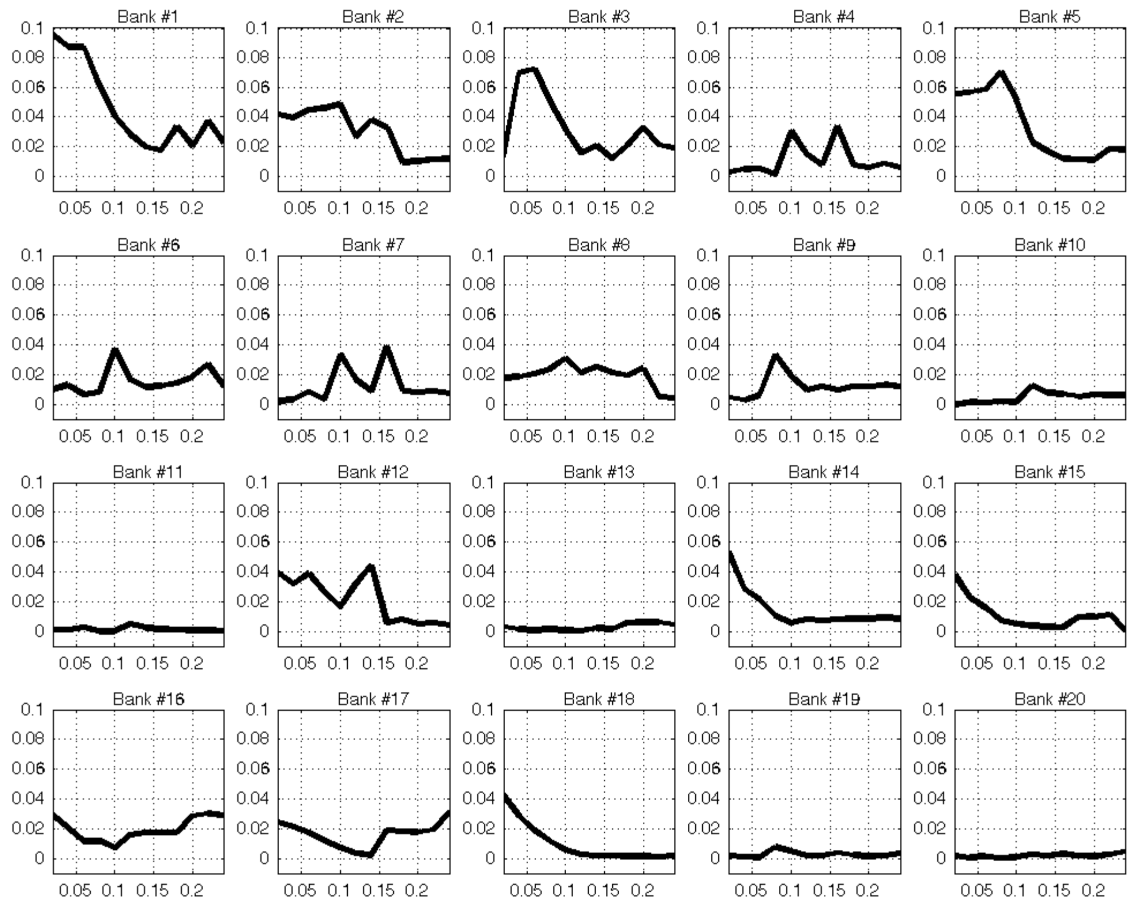


Figure 8: Contribution to systemic risk (Shapley Value, y axis) by bank for different values of γ (x axis)

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