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Critical Illness Insurance in Life Cycle Portfolio Problems

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Non-Technical Summary

The first critical illness (CI) insurance (also known as dread disease insurance) was developed in South Africa in 1983. The insurance pays a previously fixed lump sum if the insured person is diagnosed with a critical illness from a list of insured illnesses. Although the CI insurance is becoming more popular, it is still rarely used in Germany compared with disability insurance or other health-related insurance products. The relatively low demand for CI insurance is surprising due to the possible benefits. To give an example, blindness or deafness are critical illnesses that are often covered by a CI insurance. These illnesses might or might not trigger a disability insurance and lead to large costs that are not fully covered by a health insurance. A handicapped-accessible house, books for blind persons, or a special computer produce large costs. Since the expected lifetime is usually not reduced, this messes up the financial planning. I model such illnesses with health shocks in the model. Cancer or a heart attack are examples for insurable critical illnesses that reduce the expected remaining lifetime and might or might not trigger a disability insurance as well. These illnesses also produce large costs, e.g. for medicine and health care. Mortality shocks in my model capture such illnesses. The seemingly huge benefits of the CI insurance raise the question why there is little demand for this type of insurance. To the best of my knowledge, there is no life cycle model explicitly considering such an insurance.

I consider a life cycle consumption-investment-insurance problem in continuous time. The agent has to pay exogenously determined health expenses that can jump due to a critical illness of the agent. In order to avoid the excess health expenses, the agent can contract a CI insurance. The agent receives unspanned labor income and decides about the optimal consumption, investment, and insurance strategy. The financial market consists of a riskless bond and a stock. The time of death is random. The hazard rate of death can jump due to a mortality shock. A critical illness may lead to an increased mortality risk but this is not necessarily the case. In this work, I analyze whether the agent wants to contract the CI insurance or not. Moreover, I investigate the driving factors of the resulting CI insurance demand.

The increased health expenses due to a shock have a crucial impact both for the aggregate results and for the individual results. Middle-aged agents (age 45) are more than 40% better off if they do not face jumps in the health expenses. Consequently, there is a huge demand for the CI insurance. Until the age of 50, nearly all agents contract the CI insurance, even if the insurance profit is set to 200%. With human wealth becoming less uncertain when approaching retirement, the CI insurance demand decreases since a more certain income can be better used to counter the effects of a health expense jump. However, still more than 50% of the agents contract

the insurance during retirement with an insurance profit set to 20%. Middle-aged agents are about 18% better off when having access to this insurance. Before retirement when income is uncertain, low actual health expenses and a high actual income support the insurance decision. During retirement with certain income, either a low income or low health expenses can prevent the agent from contracting the insurance. A low income volatility and a low level of risk aversion decrease the demand for the CI insurance. The insurance demand also reduces significantly if agents underestimate the health expense effect of jumps or the health jump intensity.

To summarize, I find that agents strongly benefit from the possibility to hedge jumps in the health expenses, even if the insurance is very costly. Especially before retirement nearly all agents contract the CI insurance, due to the uncertainty in income.

Critical Illness Insurance in Life Cycle Portfolio Problems

Lorenz S. Schendel*

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ABSTRACT: I analyze a critical illness insurance in a consumption-investment model over the life cycle. I solve a model with stochastic mortality risk and health shock risk numerically. These shocks are interpreted as critical illness and can negatively affect the expected remaining lifetime, the health expenses, and the income. In order to hedge the health expense effect of a shock, the agent has the possibility to contract a critical illness insurance. My results highlight that the critical illness insurance is strongly desired by the agents. With an insurance profit of 20%, nearly all agents contract the insurance in the working stage of the life cycle and more than 50% of the agents contract the insurance during retirement. With an insurance profit of 200%, still nearly all working agents contract the insurance, whereas there is little demand in the retirement stage.

KEYWORDS: Health shocks, Health expenses, Labor income risk, Stochastic mortality risk, Portfolio choice

JEL-CLASSIFICATION: D91, G11, I13

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1 Introduction

A critical illness (CI) insurance delivers a fixed payment if the insured person is diagnosed a critical illness from the list of insured illnesses. Although the CI insurance is growing in popularity, there is to the best of my knowledge no life cycle model explicitly considering such an insurance.

I consider a life cycle consumption-investment-insurance problem in continuous time. The agent has to pay exogenously determined health expenses that can jump due to a critical illness of the agent. In order to avoid the excess health expenses, the agent can contract a CI insurance. The agent receives unspanned labor income and decides about the optimal consumption, investment, and insurance strategy. The financial market consists of a riskless bond and a stock. The time of death is random. The hazard rate of death can jump due to a mortality shock. A critical illness may lead to an increased mortality risk but this is not necessarily the case. In this work, I analyze whether the agent wants to contract the CI insurance or not. Moreover, I investigate the driving factors of the resulting CI insurance demand.

The health expense effect of shocks has a crucial impact both for the aggregate results and for the individual results. Middle-aged agents (age 45) are more than 40% better off if they do not face jumps in the health expenses. Consequently, there is a huge demand for the CI insurance. Until the age of 50, nearly all agents contract the CI insurance, even if the insurance profit is set to 200%. With human wealth becoming less uncertain when approaching retirement, the CI insurance demand decreases since a more certain income can be better used to counter the effects of a health expense jump. However, still more than 50% of the agents contract the insurance during retirement with an insurance profit set to 20%. Middle-aged agents are about 18% better off when having access to this insurance. Before retirement when income is uncertain, low actual health expenses and a high actual income support the insurance decision. During retirement with certain income, either a low income or low health expenses can prevent the agent from contracting the insurance. A low income volatility and a low level of risk aversion decrease the demand for the CI insurance. The insurance demand also reduces significantly if agents underestimate the health expense effect of jumps or the health jump intensity.

The importance of the health status for investment and consumption decisions is empirically analyzed in the literature with mixed results. The studies of Rosen and Wu (2004), Berkowitz and Qiu (2006) as well as Fan and Zhao (2009) find a strong relation between health status and portfolio choice. However, Love and Smith (2010) disentangle the relation between health status and portfolio choice by analyzing which part is causal and which is due to unobserved heterogeneity. They argue that health has no significant impact on portfolio choice. Of course, one expects a strong relation between health expenses and health status. Since uncertain health expenses affect the agents optimization problem in the same way as uncertain labor income does, it is a reasonable assumption that health expenses have a significant impact on consumption and investment decisions. The relevance of unspanned labor income for portfolio choice is without doubt and among others highlighted by Viceira (2001) as well as Cocco, Gomes, and Maenhout (2005).

Some recent papers include health expenses in a portfolio choice framework. Edwards (2008, 2010) analyzes a retired investor with a Cobb-Douglas utility that depends on consumption and health. In his model, agents that are unhealthy have to purchase health. In this setup, he argues that health risk can partially explain the decrease in risky investment for older people. Davidoff (2009) considers the annuity and the long-term care insurance demand as well as the consumption decision for a retired house owner with uncertain health status in a two-period model. The paper of Pang and Warshawsky (2010) is most related to my work. They model stochastic health expenses for a retired agent in a discrete-time model and analyze the impact on the optimal stock, bond, and annuity portfolio. They show that health risk leads to less risky investment and increases the annuity demand. Yogo (2012) focuses on the retirement state as well. He allows for health expenditures as a choice variable besides consumption and investment. In a discrete-time model, the agent optimizes utility from consumption, housing, and health. In contrast to the papers mentioned in this paragraph, I do not restrict my analysis to the retirement state. Furthermore, I analyze a CI insurance as a possibility to avoid excess health expenditures.

The remaining paper is organized as follows. Section 2 introduces the model setup. In Section 3, I calibrate the health expense process and present the calibrated parameters that I use in the simulations. Section 4 analyses the health expense impact of shocks and motivates the existence of a CI insurance. The CI insurance is calibrated in Section 5. Furthermore, I present results for different values of the insurance profit. Section 6 gives several sensitivity analyses with a special focus on the difference between the real-world and model-based CI insurance demand. Finally, Section 7 concludes and presents ideas for further research.

2 Model Setup

Financial Assets and Investment Decision There are two assets in the financial market. The first one is a bond B that yields the constant risk-free rate r and the second one is a stock S with constant market price of risk λ and constant volatility σ_S. The corresponding dynamics are given by

$$dB_t = B_t r\,dt,$$
$$dS_t = S_t(r+\sigma_S\lambda)\,dt + S_t\sigma_S\,dW_t^S,$$

where $W^S = (W_t^S)$ is a standard Brownian motion. The agent continuously chooses θ_t which is the fraction of financial wealth X_t that he invests into the risky asset. The remaining part, $(1-\theta_t)X_t$, is invested into the bond. I impose short-sale constraints such that the agent is restricted to $\theta_t \in [0,1]$.

Mortality Risk The time of death, denoted by τ, is uncertain and is given by the first jump of a jump process $N^D = (N_t^D)$ in intensity (hazard rate of death) $\pi(t)$. The intensity is increasing with age and can jump due to a mortality shock that permanently increases the hazard rate of death. I interpret a mortality shock as a critical illness that highly influences mortality risk, e.g. cancer. The time-dependent part of the mortality risk is modeled with a Gompertz structure. The hazard rate of death is given by

$$\pi(t) = \frac{1}{b}\mathrm{e}^{\left(\frac{t-m}{b}\right)} + \beta_\pi(t)N_t^\pi,$$

where $N^\pi = (N_t^\pi)$ is a jump process with intensity $\kappa_\pi(t)$ and is independent of all other sources of risk. In the model, N^π is allowed to jump only once. Hence, the intensity $\kappa_\pi(t)$ is set to zero after the first jump. I denote the time-dependent jump size by $\beta_\pi(t)$, whereas b and m are constant parameters that capture the increasing mortality risk over the life cycle.

Health Expenses and Insurance Decision The agent faces health expenses that are exogenously given and modeled by a geometric Brownian motion with time-dependent drift $\mu_H(t)$ and volatility $\sigma_H(t)$. The drift captures that average health expenses increase in age. The diffusive part accounts for small deviations in health expenses, e.g. induced by a common cold. Furthermore, the agent faces additional health expenses if a mortality shock or a health shock occurs. The health shock increases the health expenses significantly without increasing the mortality risk and can be interpreted as a psychical illness or a physical disability. It is modeled by the jump process $N^H = (N_t^H)$ with intensity $\kappa_H(t)$ and independently of all other sources of risk. In the model, the health expenses can jump only once. In order to hedge the health expense jump risk, the agent can contract an insurance. The insurance decision is denoted by $\iota \in \{0,1\}$. If insured, $\iota = 1$, the insurance company pays all excess health expenses due to a jump. The insurance premium depends on the actual health expense level and is denoted by $\eta(t)H_t$. The insurance decision takes place continuously and is only contracted for an interval of length dt. Thus, the health expense jump term becomes relevant only if the agent is uninsured, $\iota = 0$. The health expense dynamics are then summarized as

$$dH_t = H_t\mu_H(t)\,dt + H_t\sigma_H(t)\,dW_t^H + \mathbb{1}_{\{N_{t-}^\pi + N_{t-}^H = 0\,\wedge\,\iota_{t-}=0\}} H_{t-}\beta_H(t)\left(dN_t^\pi + dN_t^H\right), \qquad (1)$$

where $W^H = (W_t^H)$ is a standard Brownian motion that is independent of all other sources of risk. Here, $\beta_H(t)$ denotes the time-dependent health expense jump size.

Labor Income The agent receives a continuous income stream Y as long as he is alive. The income can be interpreted as labor income before retirement and pension payments after retirement. Additional to labor income uncertainty, the agent faces the risk of having to reduce work effort permanently or getting disabled due to a mortality or health shock. In this case, the income permanently reduces. The agent has no possibility to hedge the income reduction. The labor income process is allowed to jump only once. The income dynamics are given by

$$dY_t = Y_t\mu_Y(t)\,dt + Y_t\sigma_Y(t)\,dW_t^Y + \mathbb{1}_{\{N_{t-}^\pi + N_{t-}^H = 0\}} Y_{t-}\beta_Y(t)\left(dN_t^\pi + dN_t^H\right) \qquad (2)$$

with another standard Brownian motion $W^Y = (W_t^Y)$ that is independent of all other sources of risk. The income drift $\mu_Y(t)$, volatility $\sigma_Y(t)$, and jump magnitude $\beta_Y(t)$ are allowed to be time-dependent.

Preferences The agent gains utility from intermediate consumption c and terminal wealth X_τ. The utility has a constant relative risk aversion with risk aversion parameter γ. The time preference rate is given by δ and the weight of the bequest motive is denoted by ε. Hence, lifetime utility at time t is given by

$$\mathrm{E}_{t,x,y,h,A}\left[\int_t^\tau \mathrm{e}^{-\delta(u-t)}\frac{c_u^{1-\gamma}}{1-\gamma}\,du + \varepsilon\,\mathrm{e}^{-\delta(\tau-t)}\frac{X_\tau^{1-\gamma}}{1-\gamma}\right],$$

where A is a state variable that captures the current health status of the agent. A is determined by the jump processes and defined as follows:

$$A_t = \begin{cases} 1 & \text{(healthy)} \quad \text{if } N_t^H = 0 \wedge N_t^\pi = 0 \wedge N_t^D = 0, \\ 2 & \text{(health shock)} \quad \text{if } N_t^H = 1 \wedge N_t^\pi = 0 \wedge N_t^D = 0, \\ 3 & \text{(mortality shock)} \quad \text{if } N_t^\pi = 1 \wedge N_t^D = 0, \\ 4 & \text{(dead)} \quad \text{if } N_t^D = 1. \end{cases}$$

A healthy agent, $A=1$, faces health shock risk, mortality shock risk and death shock risk. Agents that only faced a health shock, $A=2$, have mortality shock risk and death shock risk, further health shocks cannot occur. If an agent suffered a mortality shock, $A=3$, he only faces risk of dying since further health or mortality shocks are not possible.

Financial Wealth and the Optimization Problem The financial wealth of the agent is denoted by X. The following wealth dynamics arise from the above model setup

$$dX_t = \left[X_t(r+\lambda\sigma_S\theta_t)+y_t-c_t-h_t-\mathbb{1}_{\{N_t^\pi+N_t^H=0\wedge\iota_t=1\}}h_t\eta(t)\right]dt+X_t\sigma_S\theta_t\,dW_t^S. \tag{3}$$

The agent maximizes lifetime utility from consumption and terminal wealth. The optimization problem is characterized by the control variables consumption c, portfolio holdings θ, and the insurance decision ι. The state variables are time t, financial wealth x, income y, health expenses h, and the health state of the agent A. The optimization problem is expressed as

$$\begin{aligned}&\max_{\{c_u,\theta_u,\iota_u\}_{u\in[0,\tau)}} \quad \mathrm{E}_{0,x,y,h,A}\left[\int_0^\tau \mathrm{e}^{-\delta u}\frac{c_u^{1-\gamma}}{1-\gamma}du+\varepsilon\,\mathrm{e}^{-\delta\tau}\frac{X_\tau^{1-\gamma}}{1-\gamma}\right]\\ &\text{s.t.}\quad dX_t=\left[X_t(r+\lambda\sigma_S\theta_t)+y_t-c_t-h_t-\mathbb{1}_{\{A_t=1\wedge\iota_t=1\}}h_t\eta(t)\right]dt+X_t\sigma_S\theta_t\,dW_t^S,\end{aligned} \tag{4}$$

and also includes short-sale constraints, $\theta_t\in[0,1]$, and liquidity constraints, i.e. the optimal choice variables have to ensure $X_t>0$. I denote the corresponding value function by J:

$$J(t,x,y,h,A)=\sup_{\{c_u,\theta_u,\iota_u\}_{u\in[t,\tau)}} \mathrm{E}_{t,x,y,h,A}\left[\int_t^\tau \mathrm{e}^{-\delta(u-t)}\frac{c_u^{1-\gamma}}{1-\gamma}du+\varepsilon\,\mathrm{e}^{-\delta(\tau-t)}\frac{X_\tau^{1-\gamma}}{1-\gamma}\right]. \tag{5}$$

The Hamilton-Jacobi-Bellman (HJB) equation of the problem is given by

$$\begin{aligned}\delta J=\sup_{c,\theta,\iota}\quad &\left\{\frac{c^{1-\gamma}}{1-\gamma}+J_t+J_x\left[(r+\lambda\sigma_S\theta)+y-c-h-\mathbb{1}_{\{A=1\wedge\iota=1\}}h\eta\right]+\frac{1}{2}J_{xx}x^2\sigma_S^2\theta^2\right.\\ &+J_y y\mu_Y+\frac{1}{2}J_{yy}y^2\sigma_Y^2+J_h h\mu_H+\frac{1}{2}J_{hh}h^2\sigma_H^2\\ &+\mathbb{1}_{\{A=1\}}\kappa_H\left[J(t,x,(1+\beta_Y)y,(1+\mathbb{1}_{\{\iota=0\}}\beta_H)h,2)-J(t,x,y,h,A)\right]\\ &+\mathbb{1}_{\{A=1\vee A=2\}}\kappa_\pi\left[J(t,x,(1+\mathbb{1}_{\{A=1\}}\beta_Y)y,(1+\mathbb{1}_{\{A=1\wedge\iota=0\}}\beta_H)h,3)-J(t,x,y,h,A)\right]\\ &\left.+\pi\left[J(\tau,x,y,h,4)-J(t,x,y,h,A)\right]\right\},\end{aligned} \tag{6}$$

for $A=\{1,2,3\}$. Subscripts of J denote partial derivatives, for example $J_t=\frac{\partial J}{\partial t}$. I solve the optimization problem numerically. An outline of the solution method is described in Appendix A.

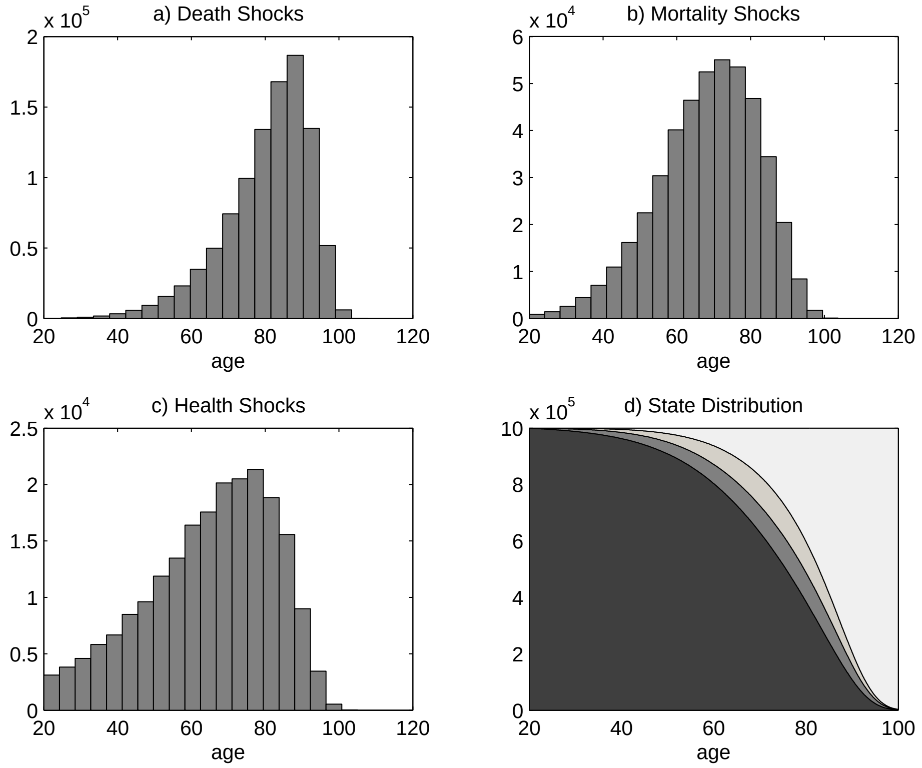

Figure 1: Shock Distribution. The figure depicts the histogram of the shocks after 1 000 000 simulations and the corresponding state distribution. a) shows the death shock distribution. b) depicts the histogram of the mortality shocks. c) presents the health shock distribution. d) depicts the resulting state distribution over the lifetime. The areas from bottom left to top right are explained as follows: The dark area corresponds to healthy agents ($A = 1$), the dark grey area to agents that faced a health shock ($A = 2$), the light grey area represents agents that faced a mortality shock ($A = 3$), and the light area indicates dead agents ($A = 4$). The processes are calibrated as stated in Section 3.

3 Calibration

Financial Assets My financial market calibration is based on Munk and Sørensen (2010). I set the risk-free rate to $r = 0.02$. I calibrate the stock with a market price of risk of $\lambda = 0.2$ and I set the volatility parameter to $\sigma_S = 0.2$.

Mortality Risk I use the mortality process and mortality shock calibration of Kraft, Schendel, and Steffensen (2014). They interpret the mortality shock as critical illness as well and calibrate it with cancer data for Germany. The intensity and magnitude of the mortality shock are given by

$$\kappa_\pi(t) = 0.02489\,\mathrm{e}^{\left(\frac{\min(t,65)+66.96}{29.42}\right)^2},$$
$$\beta_\pi(t) = 0.048 + 0.0008t.$$

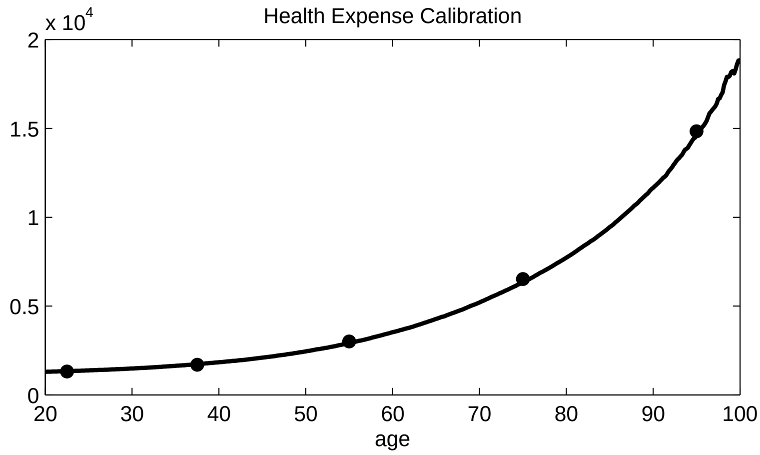

Figure 2: Health Expense Calibration. The figure compares the average health expenses after 1 000 000 simulations (line) with the data for Germany (points). The calibration for the simulated results is stated in Section 3. The data points represent average health expenses in Germany provided by Statistisches Bundesamt. The data yields average costs for the age intervals 15-29,30-44,45-64,65-84, and 85+. I draw the data points in the middle of the corresponding interval and assume a length of 20 years for the last interval. The insurance is not used here ($\iota_t = 0\,\forall t$).

The mortality process is calibrated with mortality data for Germany. The results for the parameters are $b = 6.5$ and $m = 69.45$. Figure 1 depicts the resulting death shock and mortality shock distribution after 1 000 000 simulations.

Health Expenses I calibrate the health expenses using data for Germany.[1] The data provides the average medical expenses in 2008 for six age groups. I calibrate the health jump together with the health expense drift and diffusion such that the resulting health expenses match the data. Since I do not have data for very old agents, I assume that the health expense pattern remains unchanged for agents that are older than 100. In order to simplify notation, I set: $\tilde{t} = \min(t, 80)$. I calibrate the health jump intensity and magnitude according to

$$\kappa_H(t) = \frac{1}{20.5} \mathrm{e}^{\left(\frac{\tilde{t}-88.45}{20.5}\right)},$$
$$\beta_H(t) = 6\left(\mathrm{e}^{0.01\tilde{t}}\right)^{-1} + 0.08\tilde{t} - 0.0008\tilde{t}^2.$$

The drift and volatility of the health expense process are calibrated with

$$\mu_H(t) = 0.0055 + 0.0004\tilde{t} - 0.0000137\tilde{t}^2 + 0.000000187\tilde{t}^3,$$
$$\sigma_H(t) = 0.03.$$

[1] The German health expense data is taken from “Statistisches Bundesamt, Wirtschaft und Statistik Juli 2011, p. 666”, available online at: `https://www.destatis.de/DE/Publikationen/WirtschaftStatistik/Monatsausgaben/WistaJuli11.pdf`, last access: January 21, 2014.

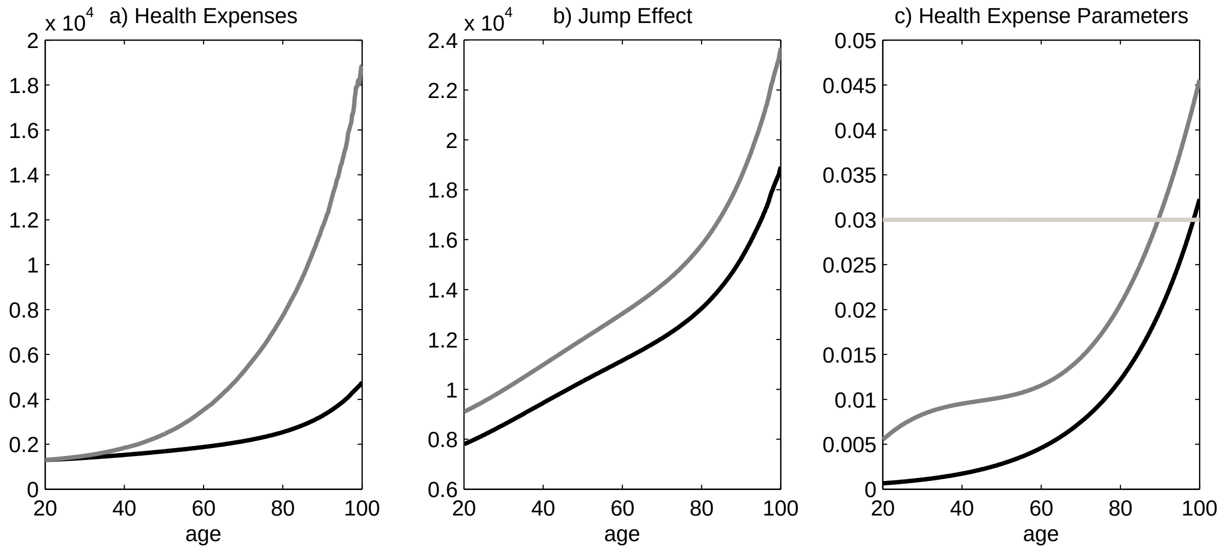

Figure 3: Health Expenses, Parameters and the Jump Effect. a) depicts the average health expenses after 1000000 simulations if shocks have no impact on the health expenses (i.e. $\beta_H = 0$) $\tilde{H}$ (dark line) and compares it to the benchmark case including the health expense impact of the shocks H (grey line). b) presents the effect of a health jump. The dark line is the average jump size $\tilde{H}\beta_H$ after 1000000 simulations and the grey line gives the corresponding average health expenses immediately after a jump $\tilde{H}(1+\beta_H)$. c) depicts the health expense parameter calibration: the jump intensity κ_H (dark line), the drift parameter μ_H (grey line), and the volatility σ_H (light line). The corresponding calibration used in the three graphs is given in Section 3. The insurance is not used here ($\iota_t = 0\,\forall t$).

I set the initial value to $H_0 = 1300$ which is a EUR 2008 value. Figure 2 compares the simulated health expenses with the above calibration to the German health expense data. The figure highlights that the simulated health expenses fit the data well. Figure 3 a) compares the average health expenses with and without the health expense effects of the shocks if the agents are not insured. The huge difference stresses the importance of the health expense jump for the agents. The difference between those lines would be captured by the insurance, if contracted. b) depicts the average jump size and the average health expenses after a jump. Intuitively, they are increasing with age. c) depicts the health jump intensity and health expense parameters over the lifetime. Figure 1 c) shows the resulting health shock distribution after 1000000 simulations. In the sample, 21.1% of the agents face a health shock. Furthermore, 45.6% of the population suffer a mortality shock that also increases health expenses for agents that had no health shock before. Figure 1 d) depicts the resulting state distribution over the life cycle. We see that most agents are either healthy or dead and only few are at the states with high health expenses at the same time.

Labor Income I calibrate the drift of the income process as in Munk and Sørensen (2010). They use PSID (Panel Study of Income Dynamics) data that yields the income dependent on the education level. The drift polynomial was originally estimated by Cocco, Gomes, and Maenhout (2005) for a discrete-time setup. They assume that the agent

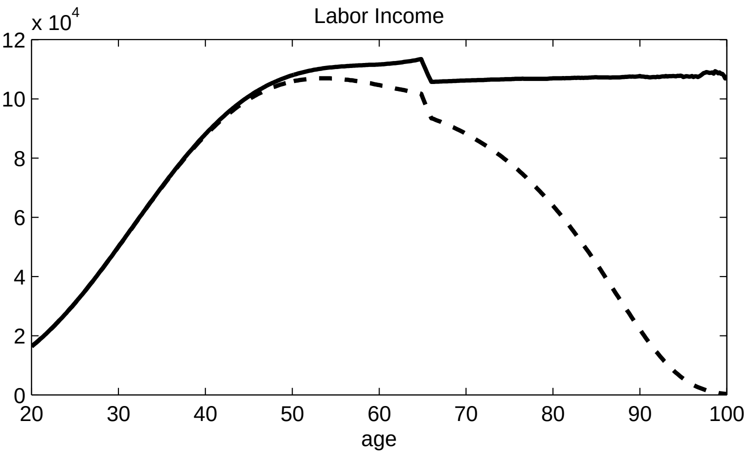

Figure 4: Expected Labor Income over the Life Cycle. The figure depicts the expected labor income profile over the life cycle after 1 000 000 simulations with the calibration given in Section 3. The solid line represents the earnings profile of all living agents, whereas the dashed line denotes the expected earnings of all agents (i.e. dead agents are included with zero income).

retires at the age of 65 and the drift is set to zero afterwards. The retirement income is a fraction of the last income before retirement. I use the continuous-time version of Munk and Sørensen (2010) which smooths the income reduction at retirement. The drift for the college education level is then given by

$$\mu_Y(t) = \mathbb{1}_{\{t<45\}}\left(0.3394 - 0.01154t + 0.000099t^2\right) - \mathbb{1}_{\{45\leq t\leq 46\}}0.06113.$$

For the diffusive component, I assume that retirement income is riskless in contrast to labor income before retirement. The diffusive component is calibrated by

$$\sigma_Y(t) = \mathbb{1}_{\{t<45\}}0.15.$$

In the case that a health jump or a mortality jump occurs, the labor income is reduced since the agent has to reduce work effort or gets disabled. I calibrate the jump size as

$$\beta_Y(t) = -\mathbb{1}_{\{t\leq 45\}}0.2.$$

Hence, the labor income reduces by 20% if the agent is still working. In contrast, the income remains unaffected if the agent is already retired. Last, I calibrate the initial value Y_0. Munk and Sørensen (2010) give a starting value for the college calibration of 13 912 USD in 2002. To be consistent with the health expense calibration, I translate the value to a 2008 value in EUR. In order to do this, I use the average EUR-USD closing mid exchange rate (source: WM/Reuters via Datastream) in 2002. Afterwards, I assume that the average income change follows the German consumer price index for the corresponding years (source: Statistisches Bundesamt)[2]. This results in $Y_0 = 16\,369$. Figure 4 depicts the average earnings profile over the life cycle.

Preferences I use standard values for the relative risk aversion $\gamma = 4$, the time preference rate $\delta = 0.03$, and the weight of the bequest motive $\varepsilon = 1$. The agent starts at the age of 20 with a financial wealth equal to one year of labor income $X_0 = Y_0$.

4 Why a Critical Illness Insurance?

In this section, I justify the existence of a CI insurance in my model and comment on the situation in the real-world insurance market. The calibration highlights a huge difference in average health expenses depending on whether shocks have an impact on the health expenses ($\beta_H \neq 0$) or not ($\beta_H = 0$) . Now, I analyze the impact of the health expense effect of the shocks on the optimal controls for the aggregate results. Furthermore, I analyze the effects for an individual agent who suffers from a health shock and/or from a mortality shock.

Aggregate Results Figure 5 compares the financial wealth evolution and optimal controls in a model with and without health expense effects of jumps. We see that the existence of the health expense jump effect increases average financial wealth. The fraction riskily invested is on average smaller and the consumption is reduced in early years. The agents are afraid of the health expense effect of the shocks. Therefore, they save more, consume less, and invest less riskily. Consequently, in later years, consumption is higher due to a high amount of accumulated wealth. This explains that the average bequest is higher as well. The differences between both models diminish for old ages and vanish almost completely at the age of 100 since shocks become less important with decreasing expected remaining lifetime.

Overall, we see that the existence of the health expense impact of the health and mortality shock has a significant effect on the aggregate results. The inclusion of the health expense effect of the jumps has qualitatively the same effect as an increase in risk aversion.

Individual Results Figure 6 depicts the effects of the shocks for an individual agent without insurance. The direct effect of a health shock at the age of 50 are a decreased income and increased health expenses. The optimal reaction is less consumption and less risky investment compared to agents without shock. Both effects are due to the reduced

[2] Data available at: `https://www-genesis.destatis.de`, table code: 61111-0001, last access: January 21, 2014.

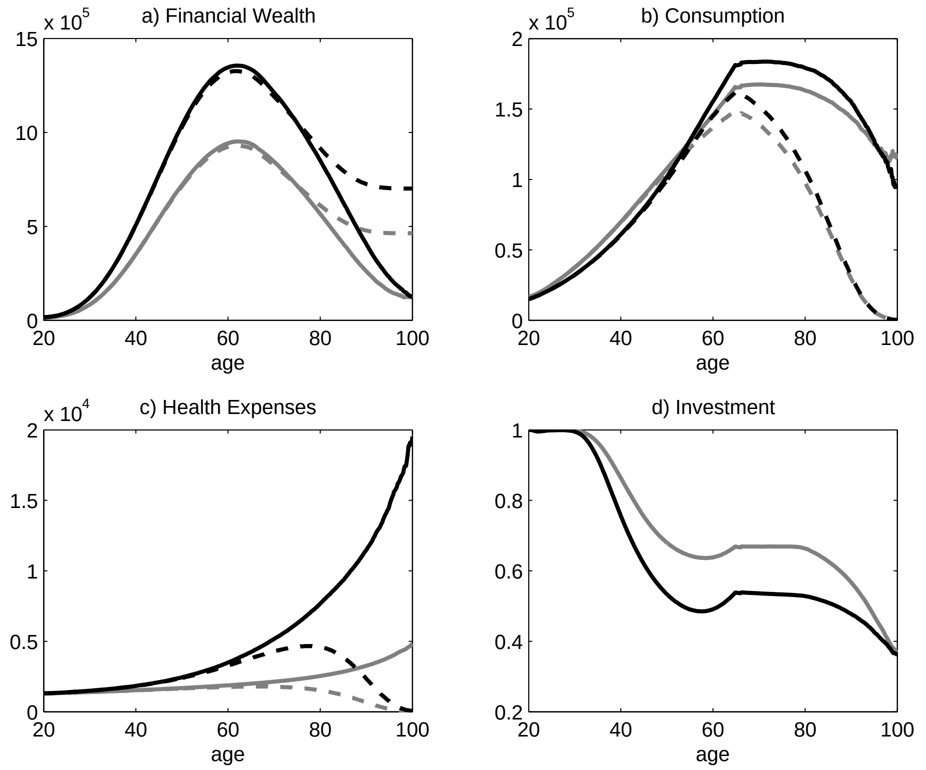

Figure 5: Aggregate Effects of Health Expense Jumps without Insurance. The figure compares the optimal controls and financial wealth evolution in two models without insurance ($\iota_t = 0\,\forall t$). The dark lines represent the benchmark model with health expense jumps, the grey lines are for a model in which shocks have no impact on health expenses, i.e. $\beta_H = 0$. The models are calibrated as stated in Section 3 and the results are averaged after 100 000 simulations. a) depicts the financial wealth evolution, b) shows consumption, c) plots the average health expenses, and d) gives the fractions of wealth invested into the risky asset. Solid lines represent results for living agents ($A \neq 4$) only and dashed lines include all agents where dead agents are included with zero consumption, zero health expenses and financial wealth equal to their bequest ($X_t = X_\tau$ if $A_t = 4$).

human wealth. The financial wealth evolution shows only a slight reduction in growth. A mortality shock at the age of 70 does not decrease income since the agent is already retired. However, it increases health expenses if there was no previous health shock. With a previous health shock, income and health expenses remain unchanged. Additionally, the mortality shock increases the hazard rate of death, which reduces the expected remaining lifetime. The optimal reaction to the mortality shock is also a decrease in the fraction of wealth that is riskily invested. The lower expected lifetime further reduces the share riskily invested as agents prefer a less risky investment for a shorter time horizon. With a previous health shock, the reduction is small and only due to the increased mortality risk. Without a previous shock, the reduction is larger and due to the reduced human wealth and the increased mortality risk. The optimal consumption increases as a result of two opposing effects. On the one hand, the increased health expenses reduce human wealth which leads to a decrease in consumption. On the other hand, the increased mortality risk increases consumption as the agent wants to spend excess wealth before his death. In this case, the mortality effect outweighs the human wealth effect. However, if a mortality

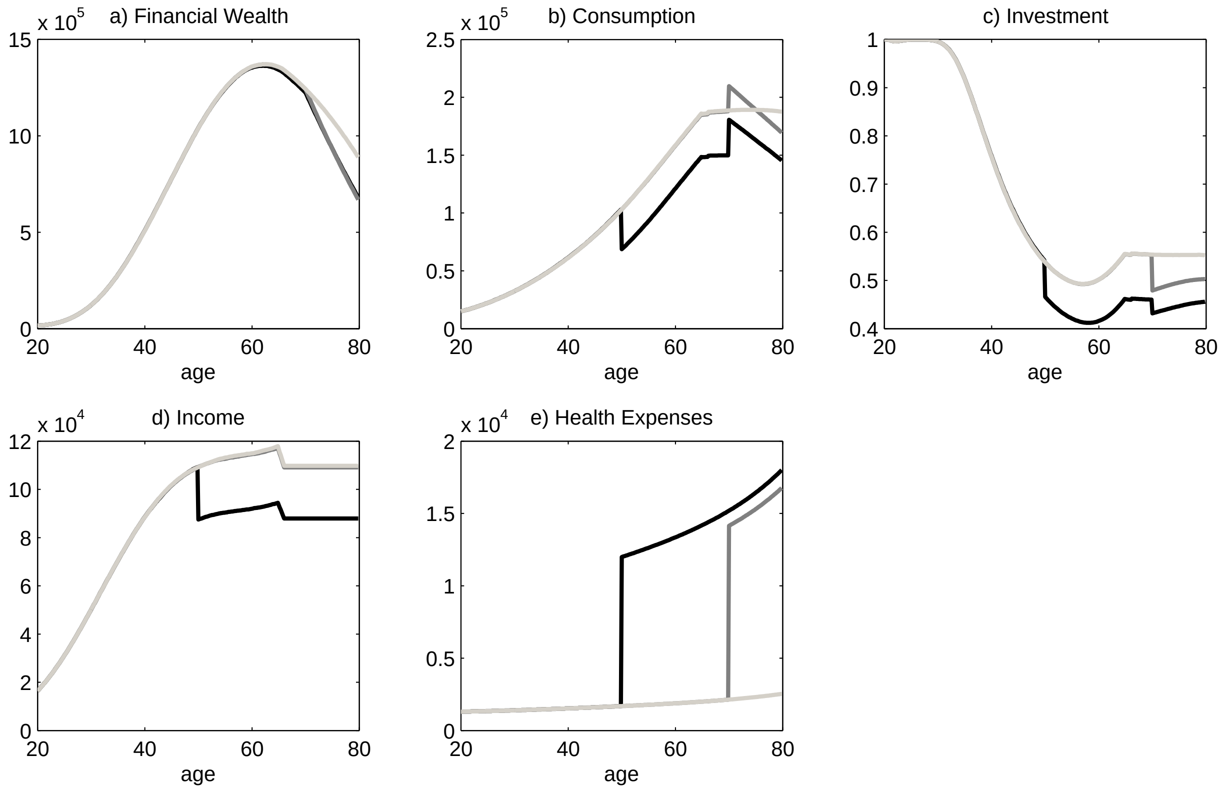

Figure 6: Sample Shocks without Insurance. The figure depicts the effects of health and mortality jumps in the lifetime, averaged from 100 000 simulations with the calibration of Section 3 without insurance ($\iota_t = 0\,\forall t$). The death shock occurs at the age of 80. The light lines show the results for no previous shocks, the grey lines are for agents with a mortality shock at the age of 70, and the agents depicted by the black lines additionally have a health shock at the age of 50. a) depicts the optimal financial wealth evolution, b) the optimal consumption, c) the optimal fraction riskily invested, d) the income over the lifetime, and e) the health expenses.

shock occurs earlier in lifetime, e.g. at the age of 50, then the human wealth effect has a larger impact and outweighs the mortality risk effect. As a result, optimal consumption would decrease as a reaction to an early mortality shock. Independent of the age, if the agent is already unhealthy and a mortality shock occurs, then consumption always increases since there is no human wealth effect and only the mortality risk effect remains. Consumption growth always decreases after a mortality shock as a result of the reduced expected lifetime. Financial wealth growth also reduces after a mortality shock because agents dissave and want to reduce accidental bequest, independent of a previous health shock. At the age of 80, the agent dies.

Figure 7 depicts the corresponding graphs without health expense effect of the shocks, i.e. $\beta_H = 0$. The health shock at the age of 50 has a less pronounced effect since the human wealth is less reduced in the absence of the health expense effect. However, consumption and risky investment is also decreased due to the income reduction. The less pronounced consumption reduction explains that financial wealth growth is slightly more reduced. The mortality shock at the age of 70 now has no human wealth effect independent of a previous

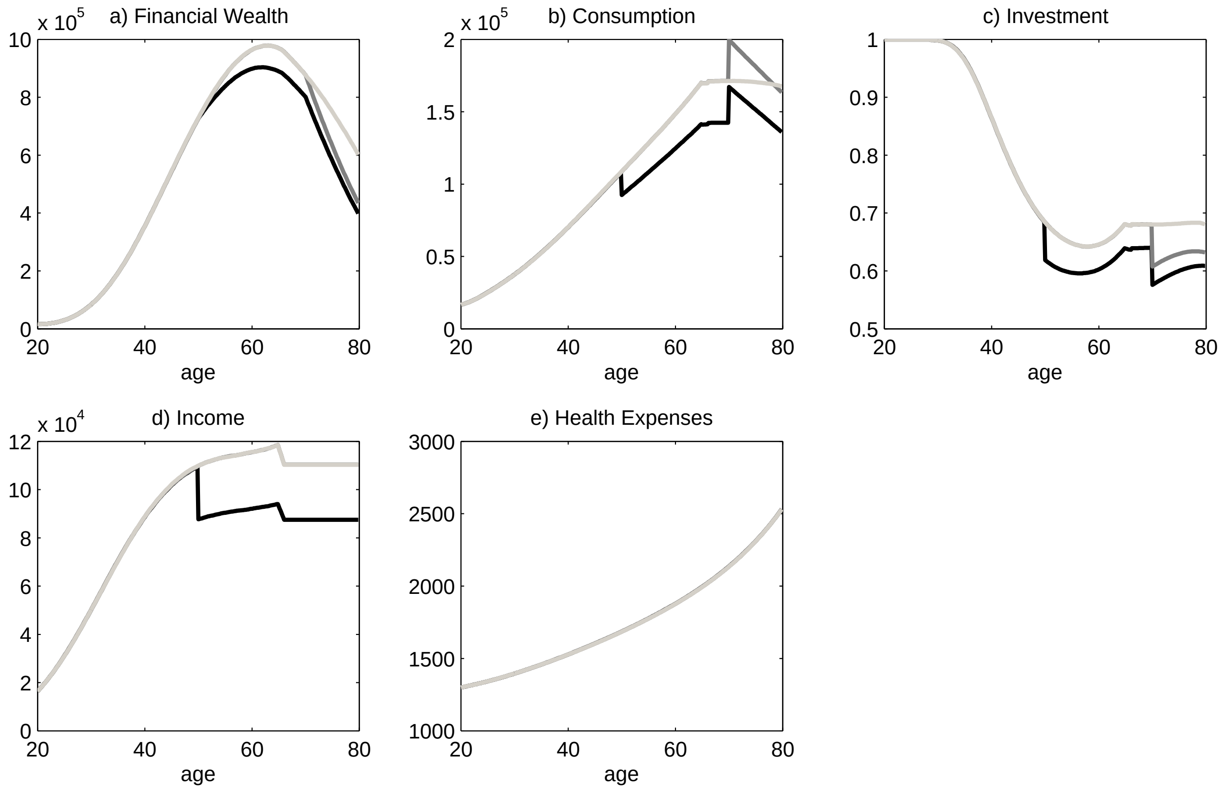

Figure 7: Sample Shocks without Health Expense Effect. The figure depicts the effects of health and mortality jumps in the lifetime, averaged from 100 000 simulations with the calibration of Section 3 without health expense effect of the shocks ($\beta_H = 0$). The death shock occurs at the age of 80. The light lines show the results for no previous shocks, the grey lines are for agents with a mortality shock at the age of 70, and the agents depicted by the black lines additionally have a health shock at the age of 50. a) depicts the optimal financial wealth evolution, b) the optimal consumption, c) the optimal fraction riskily invested, d) the income over the lifetime, and e) the health expenses.

health shock. Hence, all effects occur as an optimal reaction to the increased mortality risk and the reduced expected remaining lifetime. The shorter expected lifetime leads to less risky investment, an increase in consumption but a decrease in consumption growth and a decrease in financial wealth growth. If a mortality shock would occur before retirement, the human wealth effect is present due to the reduced income. Then, it depends on the age whether consumption increases or decreases.

Comparing Figure 6 and 7, we see that the health expense effect of the shocks is also important for the individual agent as the optimal controls differ significantly. Without health expense effect of the shocks, the human wealth effect is less pronounced which reduces the impact of the shocks for the agents.

Hence, the health expense effect of the shocks is important both for the aggregate results and for the reaction of the individual agents to the shocks. This raises the questions whether, and at which costs, agents are willing to hedge the health expense jump risk using the CI insurance.

	age 25	age 45	age 75
$CE(t,x,y,h,A)$	24.26	44.11	2.02
$CE(t,\frac{1}{2}x,y,h,A)$	24.45	46.16	2.05
$CE(t,2x,y,h,A)$	23.59	39.71	1.87
$CE(t,x,\frac{1}{2}y,h,A)$	25.83	57.71	4.71
$CE(t,x,2y,h,A)$	17.62	24.78	0.93
$CE(t,x,y,\frac{1}{2}h,A)$	17.48	23.08	0.92
$CE(t,x,y,2h,A)$	26.38	60.76	5.21

Table 1: Gain of Having no Health Expense Jumps. The table gives the percentage gain in the certainty equivalent (7) of having no health expense effect of the shocks ($\beta_H = 0$) compared to the model with health expense impact of shocks and without insurance. The percentage gain is given for a young ($t = 5$), middle-aged ($t = 25$), and old ($t = 55$) healthy ($A = 1$) agent. The other state variables are set to $x = 500\,000$, $y = 100\,000$, $h = 1\,700$. The model calibration is given in Section 3.

Welfare Impact In order to quantify the impact of the health expense effect of the shocks, I calculate a certainty equivalent which is given by

$$CE(t,x,y,h,A) = \left[\left(1-\gamma\right) J(t,x,y,h,A)\right]^{\frac{1}{1-\gamma}}. \tag{7}$$

Table 1 gives the percentage gain of having no health expense effect of the shocks for healthy agents ($A = 1$), which equals having a CI insurance for free. As expected, the gain is always positive. Considering the state variables, the age crucially influences the gain. The young agent strongly profits from having no health expense effect, whereas the middle-aged agent profits even more, but the old aged agent is only a little better off without health expense effect of the shocks. The old agent has a short remaining lifetime which damps the effect of a health expense jump. In contrast, the young agent would suffer from a health expense shock due to the long remaining lifetime but the probability of a shock in younger years is low. The middle-aged agent has a non-negligible probability for a health and mortality jump and a long enough expected remaining lifetime such that the shock has a crucial impact. The financial wealth has only a little impact on the certainty equivalent gain. For all ages, the gain increases for less financial wealth but the weak effect highlights that financial wealth is not a main driving factor. Due to the permanent effect of the health expense jump, financial wealth cannot compensate the effect, especially in early years. Variations in income have a larger impact. The more income the agent has, the less he profits from having no health expense effect of the shocks, independent of the age. A high income directly compensates high health expenses since both provide a continuous cash-flow. The older the agent is, the lower is the uncertainty with respect to future income and the better can a high income counter high health expenses. The actual health expenses have the most pronounced effect on the gain. The higher the health expenses are, the more important is the absence of the health expense effect of the shocks for the agent. The actual health expenses are particularly crucial as they directly determine the jump size. Besides, the health expense effect is similar to the income effect in the opposite direction since it provides a continuous cash-flow as well.

Thus, the health expense effect of the shocks is crucial from a qualitative and a quantitative point of view. Particularly, the young and middle-aged agents have a huge benefit from having no health expense effect of the shocks.

Actual Situation The first CI insurance (also known as dread disease insurance) was developed in South Africa in 1983.[3] The insurance pays a previously fixed lump sum if the insured person is diagnosed with a critical illness from a list of insured illnesses. The insurance is typically offered as a long-term contract. Hence, the CI insurance in my model differs from real contracts in the way that it offers a perfect hedge against the excess health expenses and is only contracted for an interval of length dt.

Although the CI insurance is becoming more popular,[4] it is still rarely used in Germany compared with disability insurance or other health-related insurance products. The relatively low demand for CI insurance is surprising due to the possible benefits. To give an example, blindness or deafness are critical illnesses that are often covered by a CI insurance. These illnesses might or might not trigger a disability insurance and lead to large costs that are not fully covered by a health insurance. A handicapped-accessible house, books for blind persons, or a special computer produce large costs. Since the expected lifetime is usually not reduced, this messes up the financial planning. I model such illnesses with the health shocks in the model. Cancer or a heart attack are examples for insurable critical illnesses that reduce the expected remaining lifetime and might or might not trigger a disability insurance as well. These illnesses also produce large costs, e.g. for medicine and health care. The mortality shocks in my model capture such illnesses.

The seemingly huge benefits of the CI insurance raise the question why there is little demand for this type of insurance. Therefore, I analyze the driving factors of the CI insurance demand.

5 Insurance Demand

In this section, I add the CI insurance to the model. I calibrate the insurance premium, present results with insurance and comment on the effects and importance of the insurance premium level.

[3] Information on real-world CI insurance contracts in this paragraph is based on: CoverTen (Incisive Financial Publishing), October 2007, available online at: `http://db.riskwaters.com/data/cover/pdf/cover_supp_1007.pdf`, last access: January 24, 2014.

[4] Estimate as of 2007, more than 20 million contracts are yearly sold worldwide (source: CoverTen, p. 13-14).

Insurance Calibration To calibrate the insurance premium, I consider 1 000 000 agents and assume that every agent is always insured. The agents are denoted with a superscript i. I calculate the average costs that occur for the insurance company corresponding to every age. I compare these costs with the income of the insurance such that the CI contract is fair for every age. For each time t, I consider all agents that are in the insurance market, i.e. agents that are healthy ($A_t^i = 1$) or face a health or mortality shock at t and were healthy before ($A_{t^-}^i = 1$). The healthy ones pay the insurance premium which yields the average insurance income (9). The agents that face a health or mortality shock at t receive a payment from the insurance. This payment is determined by the discounted difference of health expenses with and without jump effect until the time of death τ^i. This gives the average insurance outgoings (8).

$$Ins_{out}(t) = \frac{1}{\left|\{i \mid A_{t^-}^i = 1\}\right|} \sum_{i \in \{i \mid A_{t^-}^i = 1, A_t^i \neq 1\}} \int_t^{\tau^i} \mathrm{e}^{-r(u-t)} \left(H^i(u \mid \iota_{t^-}^i = 0) - H^i(u \mid \iota_{t^-}^i = 1) \right) du, \quad (8)$$

$$Ins_{in}(t) = \frac{1}{\left|\{i \mid A_{t^-}^i = 1\}\right|} \sum_{i \in \{i \mid A_t^i = 1\}} \eta(t) H^i(t). \quad (9)$$

In order to get smooth and reliable results, I consider the average income and outgoings on a yearly basis. Hence, all health, mortality, and death shocks are rounded to a full year. Now, I calibrate the insurance premium η such that the average income and outgoings are approximately identical for all ages. Resulting, I set

$$\eta(t) = \tilde{\eta} \left(0.1 + 1.227\, \mathrm{e}^{-\left(\frac{\tilde{t}-58.47}{17.35}\right)^2} + 0.936\, \mathrm{e}^{-\left(\frac{\tilde{t}-44.81}{28.05}\right)^2} \right), \quad (10)$$

where $\tilde{\eta}$ is a scaling parameter that determines the level of the insurance premium and thus, the insurance profit. For $\tilde{\eta} = 1$, the income approximately equals the outgoings such that the CI insurance is approximately actuarially fair given the insurance company faces no administrative or transaction costs. Figure 8 depicts the income and outgoings for $\tilde{\eta} = 1$.

Unfortunately, I do not have any data considering the fees and profits of CI insurance contracts. I use the average administrative fee (2.99%) and transaction fee (5.05%) given in Kraft, Schendel, and Steffensen (2014). These are the average values for German life insurance companies in 2011. Furthermore, I add the average equity return in 2011 (11.9%) to account for the insurance profit.[5] Resulting, I set $\tilde{\eta} = 1.2$ in the benchmark calibration such that the insurance company has an average profit of 20% (excluding fees). Additionally, I present results for a more expensive insurance with an average profit of 100% ($\tilde{\eta} = 2.0$) and 200% ($\tilde{\eta} = 3.0$).

[5] The average equity return is taken from the 20 biggest international insurance companies. Source: `http://www.presseportal.de/pm/39565/2367580`, last access: January 25, 2014.

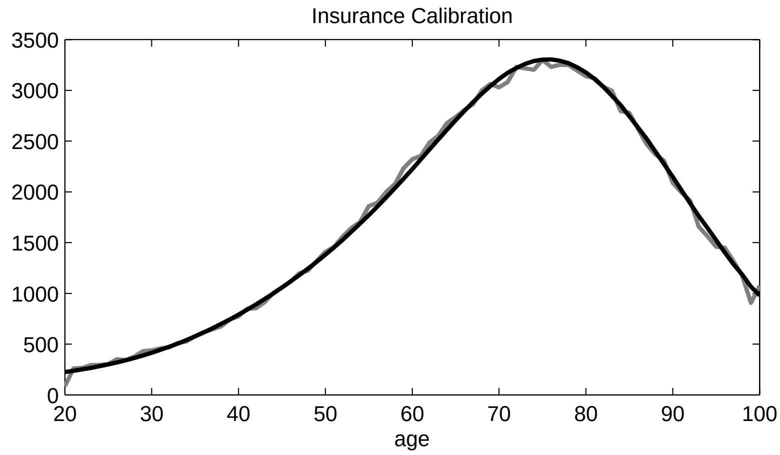

Figure 8: Insurance Calibration. The figure compares the average earnings and expenditures of the insurance company given that all agents are always insured. The grey line represents the average expenditures of the insurance calculated by (8). The dark line depicts the average earnings of the insurance calculated by (9) where η is calibrated according to (10) with $\tilde{\eta} = 1$. The model calibration is given in Section 3. The calibration is based on 1 000 000 simulations.

Benchmark Results Figure 9 depicts the results in the benchmark calibration and for more expensive CI insurance contracts. I compare the figure with Figure 5, which can be interpreted as comparing an insurance for free ($\tilde{\eta} = 0$, grey lines) and an infinitely costly insurance ($\tilde{\eta} = \infty$, dark lines). In the first case, there is no health expense effect since it is always optimal for all agents to contract the CI insurance. In the latter case, no agent can afford contracting the insurance. Hence, the setup is equal to the absence of the insurance. Consequently, I expect the results for $\tilde{\eta} \in \{1.2, 2.0, 3.0\}$ being in between those for $\tilde{\eta} \in \{0, \infty\}$. The financial wealth, consumption, and investment graphs in Figure 9 show that this is the case. The higher the insurance premium is, the more risk averse the agent behaves. Thus, he has more financial wealth, less risky investment, less consumption early in the life cycle, and more consumption later in lifetime.

Comparing the insurance decisions for the different premiums, we intuitively see that the higher the insurance premium is, the less agents contract the insurance. Considering the insurance decision in detail, we notice that nearly all agents are insured until the age of 50 independent of the insurance premium level. Then, the demand for a CI insurance decreases rapidly which is due to less uncertainty in human wealth when approaching retirement. In the benchmark calibration, the median agent is insured after retirement, whereas the median agent for the more expensive insurance is not insured anymore. For very old ages, there are only few agents in the insurance market such that the CI insurance demand is not that accurate any more. We observe that the insurance demand approaches similar levels for the different insurance premiums. Hence, the demand is less dependent on the premium level for very old ages.

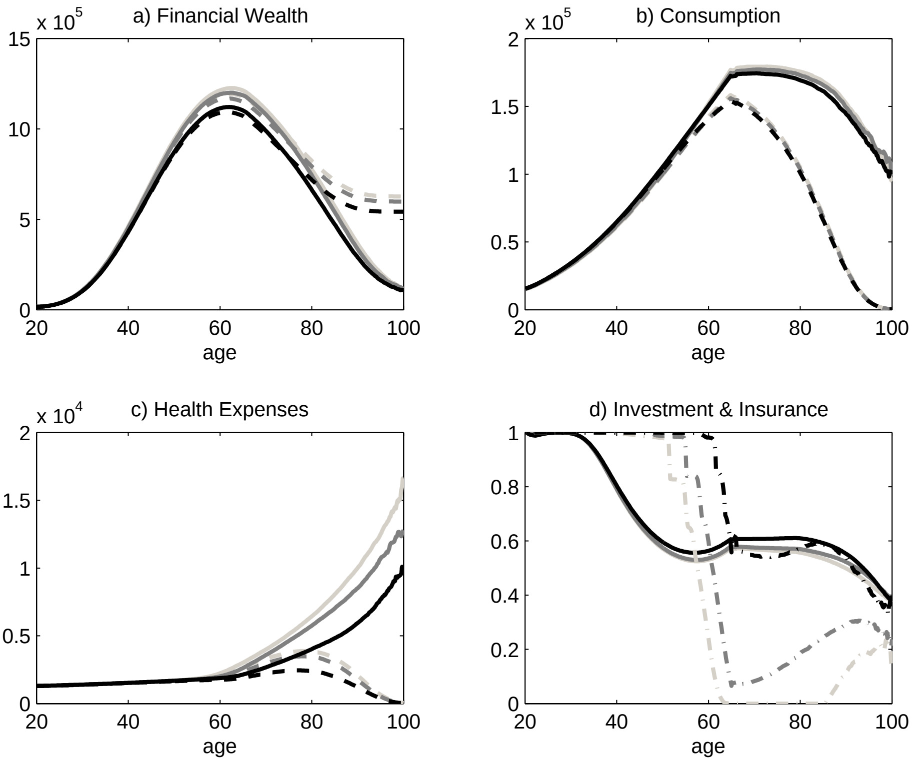

Figure 9: Results for Different Insurance Premiums. The figure depicts the average optimal controls as well as the average financial wealth and health expense evolution for different insurance premiums. The insurance premium is determined by (10). The dark lines show results for $\tilde{\eta} = 1.2$, the grey lines for $\tilde{\eta} = 2.0$ and the light lines for $\tilde{\eta} = 3.0$. a) presents the financial wealth evolution, b) gives the corresponding optimal consumption, and c) depicts the health expenses. d) shows the optimal fraction of risky investment as well as the fraction of agents that is in the insurance market ($A = 1$) and contracts a CI insurance (dash-dotted lines). Solid lines represent results for living agents ($A \neq 4$) only and dashed lines include all agents, whereas dead agents are included with zero consumption, zero health expenses, and a financial wealth equal to their bequest ($X_t = X_\tau$ if $A_t = 4$). The results are based on 100 000 simulations with the model calibration given in Section 3.

Policy Functions Next, I consider policy functions for the CI insurance demand in the benchmark case with $\tilde{\eta} = 1.2$ to analyze the impact of the state variables on the insurance decision. Figure 10 depicts the corresponding graphs. The dark grey area indicates that the agent optimally contracts the insurance, whereas he optimally has no insurance protection in the light grey area. We see that the policy functions of the young and middle-aged agent look similar, whereas the policy functions of the old agent show a completely different pattern. Considering the young and middle-aged agents in detail, the graphs for fixed health expenses depict that both income and financial wealth have no crucial impact on the CI insurance decision. The graphs for fixed income highlight that unreasonable high health expenses would be necessary such that it would be optimal not to contract the insurance and little financial wealth further supports this. In this case, the insurance comes at too high costs for the agent and is therefore not optimal. Since the contract is based on the actual health expenses, the agent cannot or does not want to afford the contract as it becomes to expensive. The graphs for fixed financial wealth

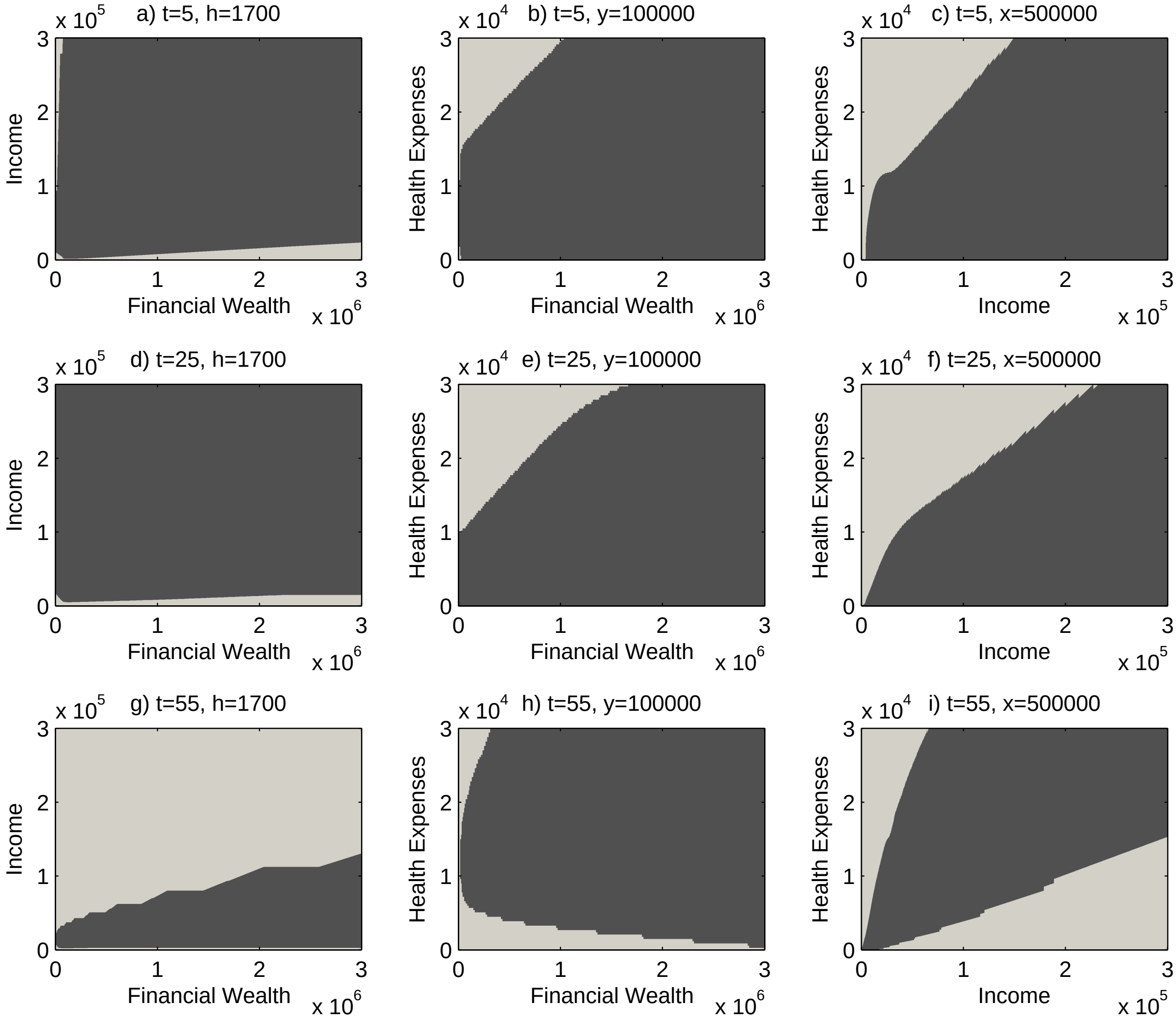

Figure 10: Policy Functions for CI Insurance Demand. The figure depicts policy functions for the CI insurance demand with the benchmark calibration in Section 3 and the insurance premium given in (10) with $\tilde{\eta} = 1.2$. The first row (a,b,c) gives policy functions for a young agent ($t = 5$), the second row (d,e,f) for a middle-aged agent ($t = 25$), and the last row (g,h,i) for an old agent ($t = 55$). The agents are healthy ($A = 1$). The first column (a,d,g) shows the policy functions for fixed health expenses with $h = 1\,700$, the second column (b,e,h) for fixed income with $y = 100\,000$, and the last column (c,f,i) for a fixed financial wealth of $x = 500\,000$. The dark grey area indicates that the agent contracts the CI insurance, whereas the light grey area indicates that the agent does not contract the insurance.

deliver the same pattern. If health expenses are extremely high and income is low, the agent does not contract the insurance since a contract becomes to expensive. Comparing the young and middle-aged agent, we see that the no-contract area increases as the age increases.

For the old agent, income has an increased importance since it is certain now. The graph with fixed health expenses highlights the importance of the income, whereas the actual financial wealth is still of little importance. With a lot certain income, the agent does not contract the CI insurance since he is able to pay the increased health expenses in the case of a jump. More financial wealth supports the decision to contract the insurance since

	age 25	age 45	age 75
$CE(t,x,y,h,A\,\|\,\tilde{\eta}=0.0)$	24.26	44.11	2.02
$CE(t,x,y,h,A\,\|\,\tilde{\eta}=1.2)$	10.33	18.30	0.00
$CE(t,x,y,h,A\,\|\,\tilde{\eta}=2.0)$	6.03	7.31	0.00
$CE(t,x,y,h,A\,\|\,\tilde{\eta}=3.0)$	3.68	3.02	0.00

Table 2: Gain of Having Access to the CI Insurance. The table gives the percentage gain in the certainty equivalent (7) of having access to the CI insurance for different insurance calibrations ($\tilde{\eta} \in \{0.0, 1.2, 2.0, 3.0\}$) compared to a model without CI insurance ($\iota = 0$). The table shows the percentage gain for a young ($t = 5$), a middle-aged ($t = 25$), and an old ($t = 55$) healthy ($A = 1$) agent. The other state variables are set to $x = 500\,000$, $y = 100\,000$, and $h = 1\,700$. The model calibration is given in Section 3 and the insurance calibration in (10).

it becomes affordable even with less income. Financial wealth cannot substitute income here, which is due to mortality risk. Particularly, income delivers a certain cash-flow until the time of death, in contrast to financial wealth. Hence, a high income can hedge a potential health expense jump that would also have an effect until the time of death. In contrast, the agent possibly outlives his financial wealth if he wants to counter increased health expenses using financial wealth and faces a late time of death. Considering the graph with a fixed income, health expenses either have to be low or extremely high such that no CI insurance contract is optimal. In the first case, the contract is not necessary, whereas the contract is too expensive in the second case. In between, contracting the CI insurance is the optimal decision. Again, more financial wealth supports an insurance contract. The graph for fixed financial wealth highlights the interaction of health expenses and income. On the one hand, a high income and low health expenses result in a rejection of the insurance. Since the agent is able to pay the increased expenses after a jump without problems, he avoids the costly insurance. On the other hand, high health expenses combined with a low income lead to no insurance as well. In this case, the agent cannot or does not want to afford the CI insurance since it is too expensive.

Altogether, health expenses have a crucial impact on the CI insurance decision over the lifetime. Income is also important for the insurance decision and the importance increases in age as the uncertainty of human wealth decreases. Financial wealth is comparably unimportant for the insurance decision since it cannot reliably hedge the effects of a health expense jump.

Welfare Analysis To evaluate the influence of the insurance premium on the importance of the insurance, I consider the percentage gain in the certainty equivalent when having access to the insurance as calculated in (7). Table 2 yields the corresponding results. It is intuitive that agents are less better off as the insurance premium increases. If the insurance is not for free, the old agent is not significantly better off when having access to the insurance, independent of the premium level. This is not surprising since the insurance demand is low for old agents due to no uncertainty in human wealth. For the middle-aged agent, the level of the premium matters most. In the benchmark calibration, the agent crucially benefits from having access to the CI insurance, whereas the gain is much lower with a high insurance premium. The young agent prefers to contract the insurance due to huge uncertainty with respect to human wealth. In contrast, the middle-aged agent has already less uncertainty with respect to human wealth, which decreases the need for a CI insurance. Therefore, the level of the premium becomes more important for the middle-aged agent.

Altogether, young and middle-aged agents are significantly better off when having access to the CI insurance. This statement holds for all premium levels, even if the insurance profit is set to 200%. This indicates a strong desire to hedge the health expense jump risk as long as there is uncertainty with respect to human wealth.

6 Sensitivity Analyses

In this section, I analyze how the insurance demand is influenced by important market features and characteristics of the agent. Especially, I consider the impact of the income parameters, the risk aversion, and underestimating the probability or magnitude of a jump in health expenses. I am seeking for explanations considering the difference between the high CI insurance demand in the benchmark model, and the low CI insurance demand in the real world.

Impact of the Income Volatility Since income is the major source of wealth for the agent, the income parameters are of special importance. In the previous section, the certain retirement income was considered to be an explanation for the lower insurance demand during and shortly before the retirement state. Figure 11 compares the benchmark model with certain retirement income to a model in which retirement income is also uncertain. Considering the CI insurance demand, the previous explanations get justified. With uncertain retirement income, the insurance demand is higher throughout the lifetime, in particular during the retirement period. Due to a higher uncertainty of human wealth, the income is less suitable as a buffer against a health expense jump. This increases the CI insurance demand. As a direct reaction to the higher insurance demand, the average health expenses reduce. The consumption and wealth graphs reflect the increased uncertainty with respect to the major source of wealth as well. The consumption is reduced in early years but is higher later in the lifetime. Hence, the agent saves more as a protection against a possibly low future income and spends the excess wealth when the

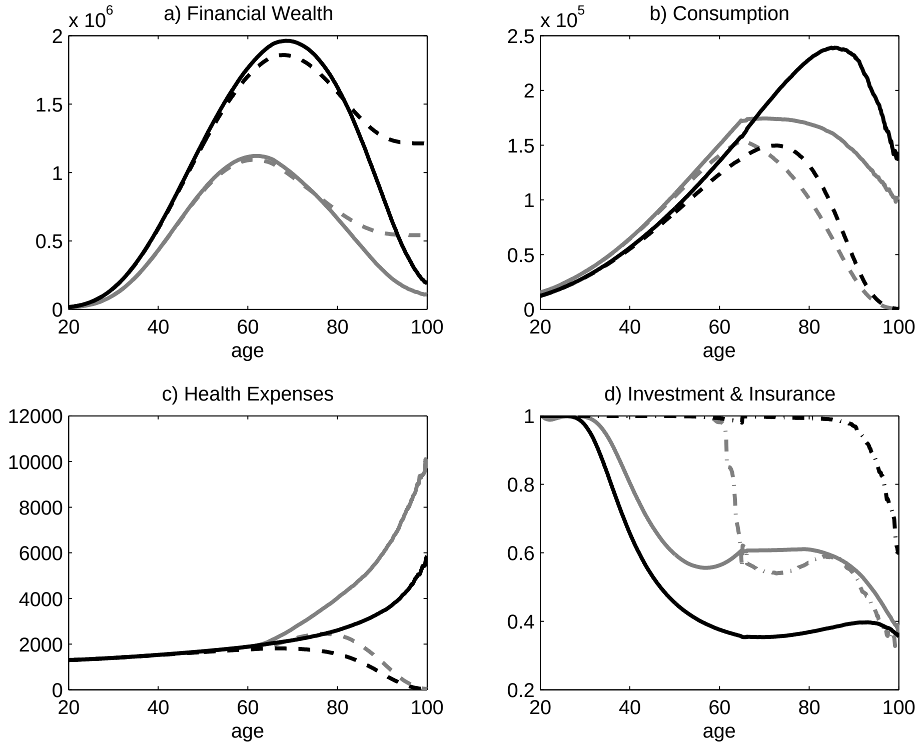

Figure 11: Impact of an Uncertain Retirement Income. The figure depicts the average optimal controls as well as the average financial wealth and health expense evolution. It compares the results with uncertain retirement income, $\sigma_Y(t) = 0.15\,\forall t$ (dark lines), to the benchmark results with certain retirement income (grey lines). a) presents the financial wealth evolution, b) gives the corresponding optimal consumption, and c) depicts the health expenses. d) shows the optimal fraction of risky investment as well as the fraction of agents that is in the insurance market ($A = 1$) and contracts a CI insurance (dash-dotted lines). Solid lines represent results for living agents ($A \neq 4$) only and dashed lines include all agents, whereas dead agents are included with zero consumption, zero health expenses, and a financial wealth equal to their bequest ($X_t = X_\tau$ if $A_t = 4$). The results are averaged based on 100 000 simulations with the model calibration of Section 3 and insurance calibration (10) with $\tilde{\eta} = 1.2$.

mortality risk increases. As a result, the agents have more financial wealth on average and leave more than twice as much bequest which is mainly accidental.

Having these results in mind, the effects of a change in the income volatility before retirement on the CI insurance demand are predictable. An increase in the volatility would further increase the insurance demand but the effect would be negligible since already nearly all agents contract the insurance early in lifetime. A reduction of the income volatility decreases the CI insurance demand before retirement. However, the income volatility is already low compared to other studies[6] such that a further reduction in income volatility is difficult to justify economically. Therefore, the income volatility cannot explain the empirically low insurance demand generally. However, it can partially explain a low insurance demand for people that have a less volatile income.

It is intuitive in the model that a reduced income volatility also reduces the CI insurance demand. In contrast, this effect is not obvious in the real world. In this paper, I model

[6] For example, Munk and Sørensen (2010) use $\sigma_Y = 0.2$ and Cocco, Gomes, and Maenhout (2005) have an overall income volatility of $\sigma_Y \approx 0.37$.

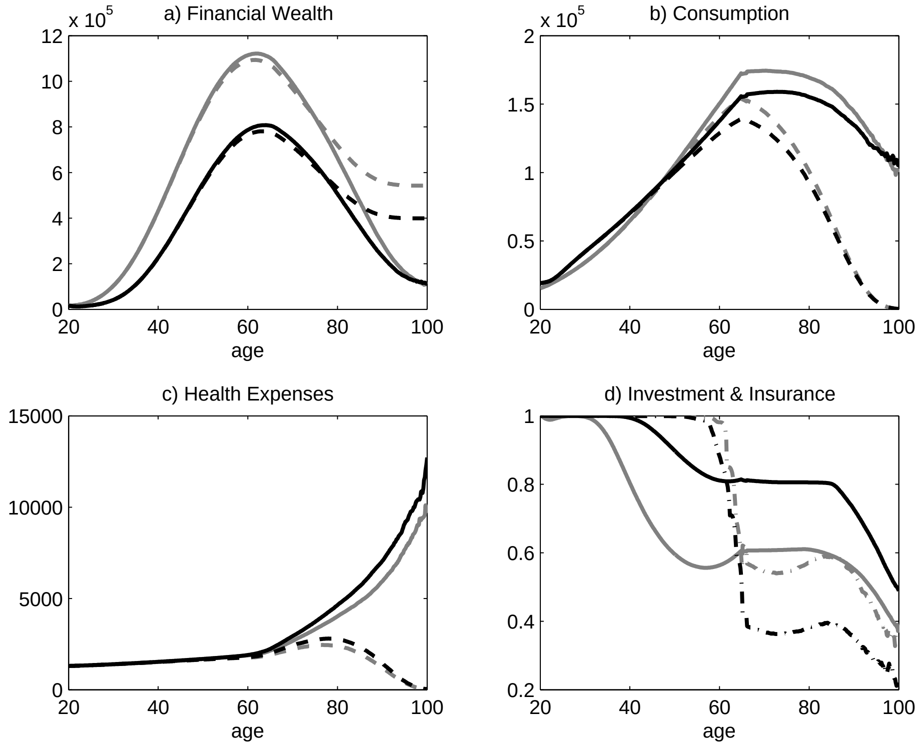

Figure 12: Impact of the Risk Aversion. The figure depicts the average optimal controls as well as the average financial wealth and health expense evolution if the agent is less risk averse, $\gamma = 3$ (dark lines), compared to the benchmark results, $\gamma = 4$ (grey lines). a) presents the financial wealth evolution, b) gives the corresponding optimal consumption, and c) depicts the health expenses. d) shows the optimal fraction of risky investment as well as the fraction of agents that is in the insurance market ($A = 1$) and contracts a CI insurance (dash-dotted lines). Solid lines represent results for living agents ($A \neq 4$) only and dashed lines include all agents, whereas dead agents are included with zero consumption, zero health expenses, and a financial wealth equal to their bequest ($X_t = X_\tau$ if $A_t = 4$). The results are averaged based on 100000 simulations with the model calibration of Section 3 and insurance calibration (10) with $\tilde{\eta} = 1.2$.

the CI insurance as an instantaneous term insurance with a contract duration of dt. However, the real-world contracts typically have a longer duration of several years. Kraft, Schendel, and Steffensen (2014) consider a model with a term life insurance and account for the typically long contract duration. In their model, the demand for term life insurance increases if the income volatility is reduced. They argue that contracting the insurance enhances the impact of a negative income evolution since a belated change of the insurance contract is costly. In an already bad state, the agent has either an undesired insurance contract or faces additional costs for changing the contract. If I modeled long-term CI insurance contracts, I would expect the same effect to take place. Then, it is unclear whether it is the fear of having an undesired contract in a bad income state or the fear of suffering a health expense jump while having a low income that dominates. However, modeling long-term CI insurance contracts would decrease the overall insurance demand. Therefore, the instantaneous term insurance modeling can serve as one explanation for the too high insurance demand in the model compared to the real world.

Impact of the Risk Aversion In previous sections, I argue that the absence of the health expense jump effect or the effect of the premium level is qualitatively equivalent to a change in risk aversion. The more expensive the CI insurance is, the more risk averse the agent behaves. In order to verify these statements, I consider the effect of a change in risk aversion in Figure 12. As noted in the previous sections, the direct effects of a reduced risk aversion are less financial wealth, less bequest, and more risky investment. The consumption is increased in early years and decreased in later years. These effects are verified in the figure. Furthermore, the figure shows that a reduced risk aversion leads to less insurance demand and therefore to higher health expenses. However, the decrease in insurance demand is low before retirement and the value of risk aversion used in the benchmark calibration is not unreasonably high. Hence, a too high risk aversion in the model compared to reality is unlikely as an explanation for the different insurance demand in the model and the real world.

Effects of Underestimating the Health Expense Jumps Another possible explanation for the low insurance demand in the real world might be that agents underestimate the probability that a shock occurs or underestimate the financial impact of a shock. To analyze this hypothesis, I consider the impact of a different belief about the health shock intensity κ_H and the health expense effect of the shocks β_H in the model. A different belief about the mortality shock intensity κ_π would also lead to a different belief about the expected remaining lifetime and is therefore not studied here.

Figure 13 depicts the effects that arise from an underestimation of the health shock intensity. The agent thinks that health shocks occur less often and thus, he expects lower average health expenses and a higher average income. Furthermore, he thinks that the insurance has a worse cost-benefit relation. Compared to the benchmark case, there is no significant effect regarding financial wealth, consumption and portfolio holdings. However, the insurance demand is reduced, especially shortly before and during retirement which yields higher health expenses.

Figure 14 depicts the effects of underestimating the health expense effect of the health and mortality shocks. The agent underestimates average health expenses and thinks that the insurance has a worse cost-benefit relation but he has a correct belief about average income. The results are similar compared to the underestimation of the intensity. There is only a little impact on financial wealth, consumption, and investment. The CI insurance demand is reduced. In particular, there is only little demand in the retirement state. However, the demand in early years is still very high. Health expenses increase again due to the reduced insurance demand.

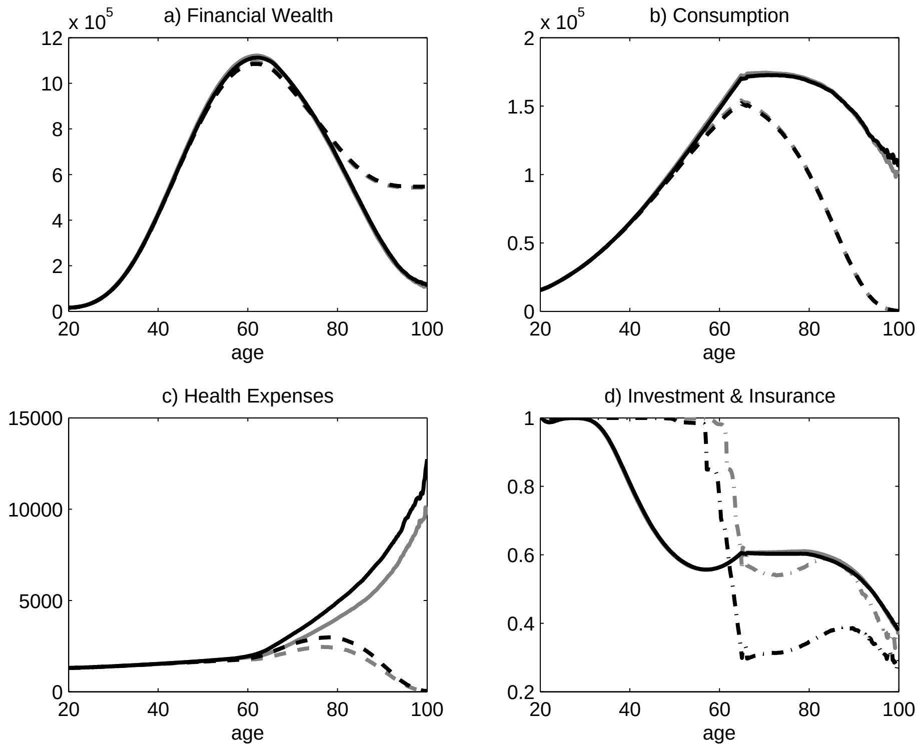

Figure 13: Impact of Underestimation of the Health Shock Intensity. The figure depicts the average optimal controls as well as the average financial wealth and health expense evolution in the benchmark model (grey lines) and in a model in which the agent underestimates the true health shock intensity (black lines). The agent thinks that health shocks occur only half as often compared to the true probability. Hence, the agent uses $\tilde{\kappa}_H(t) = 0.5\kappa_H(t)$ for his optimization, whereas $\kappa_H(t)$ is the true health jump intensity as used in the benchmark model. a) presents the financial wealth evolution, b) gives the corresponding optimal consumption, and c) depicts the health expenses. d) shows the optimal fraction of risky investment as well as the fraction of agents that is in the insurance market ($A = 1$) and contracts a CI insurance (dash-dotted lines). Solid lines represent results for living agents ($A \neq 4$) only and dashed lines include all agents, whereas dead agents are included with zero consumption, zero health expenses, and a financial wealth equal to their bequest ($X_t = X_\tau$ if $A_t = 4$). The results are averaged based on 100000 simulations with the model calibration of Section 3 and insurance calibration (10) with $\tilde{\eta} = 1.2$.

Agents that underestimate the intensity or impact of shocks are likely a partial explanation for the different insurance demand in the model and the real world. However, the CI insurance demand is mainly affected shortly before and during the retirement period.

Further Sensitivity Analyses Further sensitivity analyses cannot explain the difference between the model-based and the real-world CI insurance demand. I briefly give the results, the corresponding figures are available upon request. First, I change the preference parameters. The time preference rate δ influences the insurance demand such that for a higher time preference rate the CI insurance demand decreases. The intuition is that present cash-flows increase in value compared to future cash-flows. The CI insurance premium has to be paid when contracted, whereas the monetary benefits last throughout the lifetime if a shock occurs. Hence, the benefits decrease in value compared to the premium. For the agent, the subjective cost-benefit relation of the insurance is worse,

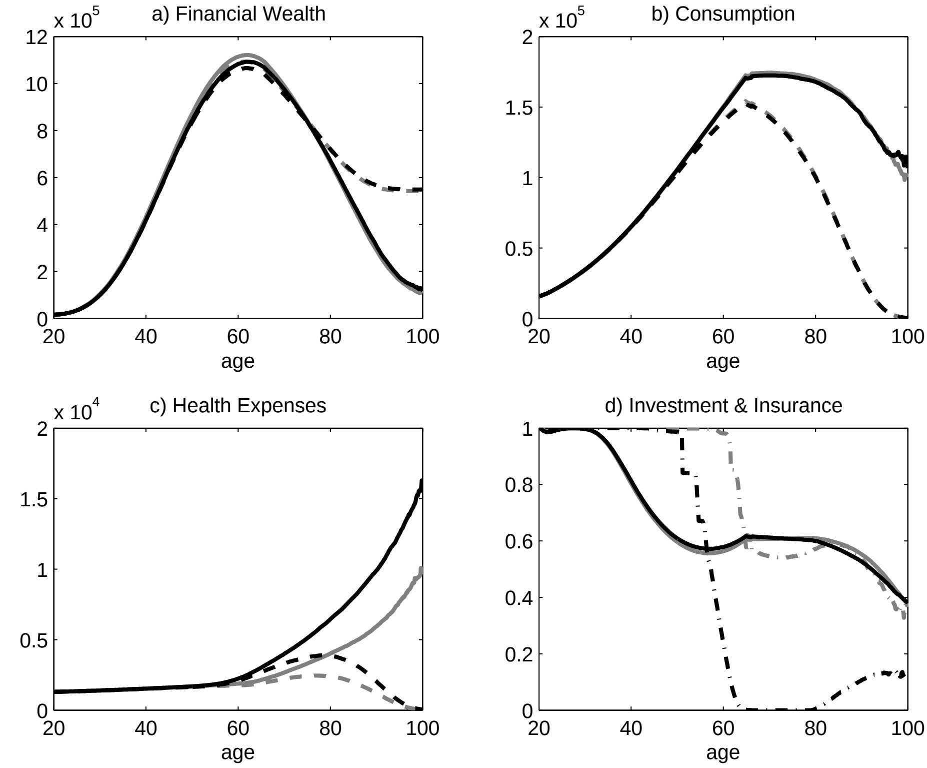

Figure 14: Impact of Underestimation of the Health Expense Jump Effect. The figure depicts the average optimal controls as well as the average financial wealth and health expense evolution in the benchmark model (grey lines) and in a model in which the agent underestimates the true health expense jump effect (black lines). The agent thinks that the health expense impact of the shocks is only half as high compared to the true impact. Hence, the agent uses $\tilde{\beta}_H(t) = 0.5\beta_H(t)$ for his optimization, whereas $\beta_H(t)$ is the true health expense effect of shocks as used in the benchmark model. a) presents the financial wealth evolution, b) gives the corresponding optimal consumption, and c) depicts the health expenses. d) shows the optimal fraction of risky investment as well as the fraction of agents that is in the insurance market ($A = 1$) and contracts a CI insurance (dash-dotted lines). Solid lines represent results for living agents ($A \neq 4$) only and dashed lines include all agents, whereas dead agents are included with zero consumption, zero health expenses, and a financial wealth equal to their bequest ($X_t = X_\tau$ if $A_t = 4$). The results are averaged based on 100 000 simulations with the model calibration of Section 3 and insurance calibration (10) with $\tilde{\eta} = 1.2$.

which yields a decreased insurance demand. Next, I consider the bequest motive and find that a change in the weight of the bequest motive ε has no significant effect on the insurance demand. In order to vary the investment opportunity set, I consider the impact of changing the stock volatility σ_S. An increase in the volatility leads to an increase in the insurance demand. The more volatile stock makes the financial wealth more volatile as well. Thus, the financial wealth is less suitable as a protection against shocks, which leads to the increased CI insurance demand. Next, I analyze the labor income drift μ_Y and the starting value Y_0. Using the high school or no high school calibration given in Munk and Sørensen (2010) instead of the college calibration, the CI insurance demand increases. The increase can be explained by the fact that the college calibration leads to an overall higher income and the retirement income is less reduced compared to the other two calibrations. Hence, the less educated agents have less human wealth which can serve as a protection against shocks. Consequently, the CI insurance demand increases. Finally, I consider a different labor income reduction if a shock occurs before retirement β_Y. An increase in β_Y means that the agent keeps more income if a shock occurs. This goes along with a decrease in the CI insurance demand before retirement since the additional income can be used to partially counter the negative effect on the health expenses. Since there is no income reduction after retirement, the effect vanishes then.

However, all of the above effects have either only a little impact or would require an unreasonable parametrization to be eligible as an explanation for the different CI insurance demand in the model and the real world.

7 Conclusion

In my model, the agents have a very high CI insurance demand, especially early in lifetime almost all agents contract the insurance. However, the CI insurance demand is significantly lower in the real world. On the one hand, the real-world CI insurance demand might be too low since it is a rather new type of insurance and it is still not very popular in most countries. On the other hand, the model-based CI insurance demand might be too high. One explanation is, that I model instantaneous term contracts which differ from real-world contracts. A long term contract, as insurance companies offer in reality, is less flexible and would therefore reduce the insurance demand. Furthermore, the insurance is modeled such that it is a perfect hedge against excess health expenses, which is also not true in reality where a fixed payout is delivered. Another explanation is that agents systematically underestimate the probability of health shocks or the magnitude of the health expense impact of the shocks. Furthermore, I do not have data on the health expense impact of the critical illnesses. Therefore, my health expense calibration might overestimate the importance of the jump part. This could also result in a too high insurance demand in the model. Despite these points, the high insurance demand, even if insurance profits are set unreasonably high, indicates that a CI insurance is worth further studies.

Further research can focus on the question why the real-world CI insurance demand is so low, despite the benefits of the insurance in the model. Modeling long-term insurance contracts would be an interesting extension to analyze the impact of the contract design in detail. Another promising extension is adding a disability insurance to analyze which type of insurance is preferred. Furthermore, a high disability insurance demand might be one explanation for the relatively low CI insurance demand in the real world.

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A The Numerical Solution Method

I solve the optimization problem (4) presented in Section 2 numerically. The numerical approach is similar to the one used in Schendel (2014) which is based on the procedure in Munk and Sørensen (2010). Details considering the numerical method are given in the papers mentioned above. For the numerical solution, I simplify the problem by reducing the number of state variables by one.

Lemma 1. *The number of state variables in the optimization problem (4) can be reduced by one. The value function (5) can be expressed as*

$$J(t,x,y,h,A) = y^{1-\gamma}F(t,\hat{x},\hat{h},A)$$

with $\hat{x} = \frac{x}{y}$ *and* $\hat{h} = \frac{h}{y}$. *The corresponding HJB of the simplified problem is given by*

$$\begin{aligned}
0 = \sup_{\hat{c},\theta,\iota} \quad & \Bigg\{ \frac{\hat{c}^{1-\gamma}}{1-\gamma} + F\left[-\delta - \pi - \mathbb{1}_{\{A=1\}}\kappa_H - \mathbb{1}_{\{A=1 \vee A=2\}}\kappa_\pi + (1-\gamma)\mu_Y - 0.5\gamma(1-\gamma)\sigma_Y^2\right] + F_t \\
& + F_{\hat{x}}\left[\hat{x}\left(r + \lambda\sigma_S\theta - \mu_Y + \gamma\sigma_Y^2\right) + 1 - \hat{c} - \hat{h} - \mathbb{1}_{\{A=1 \wedge \iota=1\}}\hat{h}\eta\right] + 0.5F_{\hat{x}\hat{x}}\hat{x}^2\left[\sigma_S^2\theta^2 + \sigma_Y^2\right] \\
& + F_{\hat{h}}\hat{h}\left[\mu_H - \mu_Y + \gamma\sigma_Y^2\right] + 0.5F_{\hat{h}\hat{h}}\hat{h}^2\left[\sigma_H^2 + \sigma_Y^2\right] \\
& + \mathbb{1}_{\{A=1\}}\kappa_H F\left(t, \left(\frac{1}{1+\beta_Y}\right)\hat{x}, \left(\frac{1 + \mathbb{1}_{\{\iota=0\}}\beta_H}{1+\beta_Y}\right)\hat{h}, A=2\right) \\
& + \mathbb{1}_{\{A=1 \vee A=2\}}\kappa_\pi F\left(t, \left(\frac{1}{1 + \mathbb{1}_{\{A=1\}}\beta_Y}\right)\hat{x}, \left(\frac{1 + \mathbb{1}_{\{A=1 \wedge \iota=0\}}\beta_H}{1 + \mathbb{1}_{\{A=1\}}\beta_Y}\right)\hat{h}, A=3\right) \\
& + \pi F\left(\tau, \hat{x}, \hat{h}, A=4\right)\Bigg\},
\end{aligned} \tag{11}$$

for $A \in \{1,2,3\}$ *where* $\hat{c} = \frac{c}{y}$. *The simplified optimization problem has only four state variables* $t,\hat{x},\hat{h},A$ *and three control variables* $\hat{c},\theta,\iota$, *whereat hat-variables are normalized by the income level. The optimal normalized consumption and optimal portfolio holdings for a given insurance decision* $\iota \in \{0,1\}$ *can be calculated according to*

$$\begin{aligned}
\hat{c} &= F_{\hat{x}}^{-\frac{1}{\gamma}}, \\
\theta &= -\frac{F_{\hat{x}}\lambda}{F_{\hat{x}\hat{x}}\hat{x}\sigma_S}.
\end{aligned}$$

With the optimal controls $\hat{c}$ *and* θ *for both possible insurance decisions,* ι *is calculated by*

$$\iota = \underset{\iota\in\{0,1\}}{\arg\max} \quad F(t,\hat{x},\hat{y},A).$$

Proof. First, I reduce the number of state variables. Due to the linearity of the financial wealth (3), income (2), and health expense (1) dynamics and the power utility setup, I can calculate for $k > 0, k \in \mathbb{R}$

$$\begin{aligned} J(t,kx,ky,kh,A) &= \sup_{\{c_s,\theta_s,\iota_s\}_{s\in[t,\tau)}} \mathrm{E}_{t,x,y,h,A}\left[\int_t^\tau e^{-\delta(s-t)}\frac{(kc_s)^{1-\gamma}}{1-\gamma}ds + \varepsilon\, \mathrm{e}^{-\delta(\tau-t)}\frac{(kX_\tau)^{1-\gamma}}{1-\gamma}\right] \\ &= k^{1-\gamma} \sup_{\{c_s,\theta_s,\iota_s\}_{s\in[t,\tau)}} \mathrm{E}_{t,x,y,h,A}\left[\int_t^\tau e^{-\delta(s-t)}\frac{c_s^{1-\gamma}}{1-\gamma}ds + \varepsilon\, \mathrm{e}^{-\delta(\tau-t)}\frac{X_\tau^{1-\gamma}}{1-\gamma}\right] \\ &= k^{1-\gamma} J(t,x,y,h,A). \end{aligned}$$

Thus, I can reduce the number of state variables by one via expressing the indirect utility as

$$\begin{aligned} J(t,x,y,h,A) &= y^{1-\gamma} J\left(t,\frac{x}{y},\frac{y}{y},\frac{h}{y},A\right) \\ &=: y^{1-\gamma} F(t,\hat{x},\hat{h},A) \end{aligned}$$

with the new introduced normalized state variables $\hat{x} = \frac{x}{y}$ and $\hat{h} = \frac{h}{y}$. I express the partial derivatives of the HJB (6) in terms of F, which yields

$$\begin{aligned} J_t &= y^{1-\gamma} F_t, \\ J_x &= y^{1-\gamma}\frac{1}{y}F_{\hat{x}}, \\ J_{xx} &= y^{1-\gamma}\frac{1}{y^2}F_{\hat{x}\hat{x}}, \\ J_h &= y^{1-\gamma}\frac{1}{y}F_{\hat{h}}, \\ J_{hh} &= y^{1-\gamma}\frac{1}{y^2}F_{\hat{h}\hat{h}}, \\ J_y &= (1-\gamma)y^{-\gamma}F - y^{-\gamma}\frac{x}{y}F_{\hat{x}} - y^{-\gamma}\frac{h}{y}F_{\hat{h}}, \\ J_{yy} &= -\gamma(1-\gamma)y^{-\gamma-1}F + 2\gamma\frac{x}{y}y^{-\gamma-1}F_{\hat{x}} + 2\gamma\frac{h}{y}y^{-\gamma-1}F_{\hat{h}} + \frac{x^2}{y^2}y^{-\gamma-1}F_{\hat{x}\hat{x}} + \frac{h^2}{y^2}y^{-\gamma-1}F_{\hat{h}\hat{h}}. \end{aligned}$$

Inserting the new value function and the above derivatives in the HJB (6) and defining the new normalized control variable $\hat{c} = \frac{c}{y}$ leads to the new simplified HJB (11). For solving the simplified optimization problem, I first consider the HJB (11) for a fixed $\iota \in \{0,1\}$. Then, I can calculate the optimal normalized consumption and the optimal portfolio holdings, conditional on ι by taking the first order conditions of the HJB (11). Next, I substitute the calculated optimal controls into the HJB. The optimal insurance decision ι is then defined as the argument that maximizes the value function. □

I solve the new simplified optimization problem numerically with an implicit finite difference backward iterative approach. I set up a grid of normalized wealth $\hat{x} \in (0, 150]$ with 1 000 grid points, of normalized health expenses $\hat{h} \in (0, 3]$ with 500 grid points, and of time $t \in [0, 110]$ with 661 grid points. I start with the solution for the case $A = 4$, which is trivial as there is no decision. Afterwards, I calculate the solution for $A = 3$, followed by $A = 2$, and lastly $A = 1$. In the state $A = 1$, I first calculate optimal normalized consumption and portfolio holdings for both possible insurance decisions. Then, I take the insurance decision that maximizes the value function calculated with the other optimal controls in each grid point. With the optimal insurance decision, I choose the corresponding optimal normalized consumption and portfolio holdings. After having calculated the optimal controls, I simulate 100 000 life cycles.

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