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Partial Information about Contagion Risk, Self-Exciting Processes and Portfolio Optimization

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Abstract

This paper compares two classes of models that allow for additional channels of correlation between asset returns: regime switching models with jumps and models with contagious jumps. Both classes of models involve a hidden Markov chain that captures good and bad economic states. The distinctive feature of a model with contagious jumps is that large negative returns and unobservable transitions of the economy into a bad state can occur simultaneously. We show that in this framework the filtered loss intensities have dynamics similar to self-exciting processes. Besides, we study the impact of unobservable contagious jumps on optimal portfolio strategies and filtering.

Keywords: Asset Allocation, Contagion, Nonlinear Filtering, Hidden State, Self-exciting Processes

JEL: G01, G11

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1 Introduction

One of the main contributions in finance over the last 50 years is to point out that correlations have a decisive impact on asset pricing and asset allocation: Risk premia increase with covariances between asset and market returns, optimal portfolio shares depend on correlation structures in portfolios, and prices of portfolio derivatives vary with correlation structures as well. This paper studies a special type of comovement, so-called contagion or domino effects, and their impact on portfolio decisions. Contagion refers to a situation where losses in certain assets or asset classes (e.g. bank shares, government bonds) trigger a cascade of losses in other assets or asset classes. Since contagion effects heavily influence correlations, capturing them in financial models is crucial. Several approaches to model contagion have been suggested.[1] Important contributions include (hidden) Markov chain models and self-exciting models that have recently been discussed in the literature. Typically, Markov chain models distinguish between good ('boom') and bad ('depression') states of the world, where in a bad state the probabilities and/or correlations for/between losses are higher than in good states. Additionally, some authors assume that agents are not able to observe the current state of the economy. On the other hand, self-exciting models directly allow for cascades of self-enhancing increases in loss probabilities. More precisely, initial losses temporarily increase the probability of further losses, which formally resembles the above-mentioned intuitive interpretation of contagion effects.

Our paper contributes to the existing literature in several dimensions: Firstly, we introduce a hidden Markov chain model that allows for large negative returns and (unobservable) regime shifts at the same time. This is a relevant model specification since, economically, these losses are particularly significant as they happen at the same time when economic conditions are worsening. We show that exactly this specification induces self-exciting loss intensities. Recent empirical evidence by Ait-Sahalia, Cacho-Diaz, and Laeven (2012) suggests that this model class fits stock dynamics very well. Intuitively, our model captures events such as the 'Black Thursday', October 24, 1929. On that day, U.S. markets fell by 11% at the opening bell, which marked the beginning of the Wall Street Crash in 1929.[2] The connection between self-exciting and hidden Markov chain models is a remarkable insight since it links together two model classes that, at first sight, seem to be different. From this point of view, self-exciting models can be interpreted as reduced-form versions of hidden Markov chain models. We compare this specification with other hidden Markov chain models and point out differences between the filtered loss intensities in these models

[1] A more detailed literature survey can be found below.

[2] The Dow Jones Industrial Average was 305.85 on October 23, 1929, and decreased to 198.69 on November 13, 1929.

and in our model. Furthermore, as an application we study asset allocation decisions in hidden Markov chain models. We find that optimal portfolio strategies differ significantly depending on whether a regime switching model or a model with contagious jumps is used. In particular, regime switching models lead to noisier strategies, whereas in the latter model the updates upon losses are more pronounced. In a simulation study, we evaluate the performance of several investment strategies relying on different filtering methods. The utility losses from not filtering at all can be substantial. The utility losses from using the wrong filter are moderate, but can become significant if the investment horizon is large (such as 50 years in a life-cycle setting).

There are several ways to capture contagion risk. One strand of literature models contagion as simultaneous Poisson jumps in all assets, e.g. Das and Uppal (2004). Kraft and Steffensen (2008) extend this approach to bond markets and default risk. Ait-Sahalia, Cacho-Diaz, and Hurd (2009) consider a setting with several assets. All these papers abstract from the time dimension of contagion. In particular, the probability of subsequent crashes remains the same after a joint jump. The second strand of literature are regime switching or Markov switching models. Early references in finance and economics include Schwert (1989), Turner, Startz, and Nelson (1989), and Hamilton (1989). Ang and Bekaert (2002) apply this approach to a discrete-time asset allocation problem, whereas Honda (2003) focuses on a continuous-time framework. Recent studies with different interpretations, parametrizations, and calibrations of the regimes include Kole, Koedijk, and Verbeek (2006) and Guidolin and Timmermann (2007, 2008). Although a regime switching model can capture the time dimension of contagion, regime shifts are still triggered by a process that is not linked to a particular crash in some asset. Apart from these two main ideas of modeling contagion, other approaches have been developed. For instance, Buraschi, Porchia, and Trojani (2010) focus on the impact of stochastic correlation on an optimal portfolio and suggest contagion risk as one application of their method.

Some recent papers model contagion effects more explicitly. In this respect, our paper is related to Branger, Kraft, and Meinerding (2009). They focus on model risk and show that an investor modeling contagion using joint jumps can suffer severe utility losses once he is confronted with a Markov regime-switching framework. Kraft and Steffensen (2009) develop a similar model and apply it to the bond market, but focus on a complete market only. In contrast to our paper, Branger, Kraft, and Meinerding (2009) and Kraft and Steffensen (2009) assume that investors can observe the state of the economy perfectly. Ding, Giesecke, and Tomecek (2009) and Ait-Sahalia, Cacho-Diaz, and Laeven (2012) propose a different class of stochastic processes to model contagion effects, so-called self-exciting processes (Hawkes processes). They find that these can generate the empirically observed amount of default clustering. Our paper is complementary to their studies. More precisely, we find that the filtered jump intensities in a model with contagious jumps follow self-exciting processes with state-dependent coefficients.

Our paper is related to the literature on continuous-time portfolio choice under complete information starting with Merton (1969, 1971). Early models with jump-diffusion processes have been developed by Aase (1984) and Jeanblanc-Picqué and Pontier (1990). Liu, Longstaff, and Pan (2003) consider a setup with jumps in stock prices and volatilities and solve for the optimal portfolio in an incomplete market. Liu and Pan (2003) and Branger, Schlag, and Schneider (2008) study related problems with derivatives. Wu (2003) focuses on a stochastic, but predictable investment opportunity set. Davis and Lleo (2011) study a portfolio problem allowing jumps in asset prices and factor processes, as well as stochastic volatility and investment constraints.

Methodologically, our paper also builds on the large amount of literature on learning and incomplete information. The seminal studies of Detemple (1986) and Dothan and Feldman (1986) were among the first applying filtering techniques to asset pricing and asset allocation under partial information. They show that these problems can be decomposed into two parts: First a filtering problem must be solved, i.e. the current value of the state variable is estimated. Second, conditional on the estimated state variable, the optimal portfolio is determined. In diffusion settings, Honda (2003) studies a portfolio problem with unobservable regimes and Liu (2011) generalizes his results to ambiguity averse investors. In a recent paper, Liu, Peleg, and Subrahmanyam (2010) quantify the value of information in portfolio choice within a diffusion model. Björk, Davis, and Landen (2010) generalize the mathematical framework to compute optimal investment strategies under partial information. However, they still assume that asset prices follow diffusion processes. References on incomplete information about jump processes include Brémaud (1981) and the recent papers by Frey and Runggaldier (2010) and Frey and Schmidt (2012). Bäuerle and Rieder (2007) and Callegaro, di Masi, and Runggaldier (2006) use such filters in portfolio theory. A comprehensive overview of models with incomplete information in finance is given by Pastor and Veronesi (2009).

The remainder of this paper is structured as follows. Section 2 presents the models (contagion, regime switching), the filtering equations and the link to self-exciting processes. Section 3 studies the asset allocation problems. In Section 4, we provide numerical results showing the effect of contagion risk and filtering on an investor's optimal portfolio strategy in more detail. Section 5 concludes. All proofs can be found in the Appendix.

2 Framework

2.1 Three Models

Crucial for the modeling of contagion effects is a channel through which loss dependence can temporarily spike making a sequence of losses more likely. To capture this self-enhancing effect, self-exciting (Hawkes) models postulate dynamics for loss intensities that are of the form[3]

$$d\lambda_t \quad = \quad \kappa(\theta - \lambda_t)dt + \sigma_t dW_t + \ell dN_t, \tag{1}$$

where N is a counting process with intensity λ and ℓ denotes a positive jump size. The volatility σ_t is either set to zero or chosen such that λ remains positive (e.g. $\sigma_t = \sigma\sqrt{\lambda_t}$). The main feature of this specification is that jumps (counted by N) temporarily increase the probability of subsequent jumps, but their impact fades away due to the mean reversion feature in the drift term. Put differently, without additional jumps the process reverts back to θ with speed κ.[4]

Another (more indirect) way to capture domino effects is to allow for different states of the economy ('good' and 'bad') where in bad states loss probabilities increase. Since, realistically, these states are not observable, agents must filter the current state from observable asset prices. More precisely, agents filter the probability of being in a particular state as well as jump intensities. As we will see, this also leads to implied loss intensities that can be similarly interpreted as (1). The *main goals of this section* are (i) to derive these implied loss intensities for economies where contagion is captured by one of several specifications of a hidden Markov chain model, (ii) to compare them to (1), and (iii) to highlight which of them can be interpreted as self-exciting processes.

In the standard model by Merton (1969), dependence is induced by correlated Brownian shocks. This framework is not able to produce realistic probabilities for large losses or for contagion effects.[5] In this paper, we thus consider models that involve a (hidden) Markov chain capturing the state of the economy and that allow for Poisson jumps. We show that it is crucial whether transitions of the Markov chain are connected to asset price jumps. More precisely, the following model specifications are studied:

[3]See, e.g., Ding, Giesecke, and Tomecek (2009) and Ait-Sahalia, Cacho-Diaz, and Laeven (2012).

[4]Analogously to a Vasicek model, it is common in the literature to refer to θ as the mean reversion level (see, e.g., Ait-Sahalia, Cacho-Diaz, and Laeven (2012)). However, the long run mean of λ is generally different from θ because N is not a martingale.

[5]See, e.g., Ait-Sahalia, Cacho-Diaz, and Laeven (2012).

(a) In a *pure regime switching model*, asset prices follow diffusion processes conditioned on the state of the economy (see, e.g., Honda (2003)). Such a model allows for additional dependence compared to the standard model since drift and diffusion parameters can depend on the state.

(b) In a *regime switching model with jumps*, asset prices follow jump-diffusion processes conditioned on the state of the economy. The model allows for jumps within a state and it is also possible that jump probabilities change across states (see, e.g., Bäuerle and Rieder (2007)). This can induce additional dependence compared to the model studied by Honda (2003).

(c) In a *model with contagious jumps*, it is additionally possible that some of the jumps in a good state induce a regime shift to a bad state.[6] Therefore, this model allows for jumps within good and bad states, but also for jumps upon transition from a good to a bad state. A risk averse agent particularly dislikes the latter jumps since declining economic conditions (via increasing probabilities of future losses) go together with current losses. We refer to these jumps as contagious.

To highlight the main ideas, we describe the differences between the frameworks for a simple economy where the investor can invest in a money market account and two risky assets A and B.[7] The dynamics of the money market account are given by

$$dM_t = rM_t dt, \quad M_0 = 1,$$

where, for simplicity, the interest rate r is assumed to be constant. The state of the economy is described by a Markov chain that can be in one of two states, a good or calm state ('calm') and a bad or contagion state ('cont'). This is captured by the process p_t that is 1 in the calm state and 0 in the contagion state. Its dynamics are given by

$$dp_t = dN_t^0 - dN_t^1,$$

where N^0 and N^1 are counting processes that trigger jumps into the good or bad state, respectively. The corresponding intensities are defined by $\lambda_t^0 = (1 - p_t)\lambda^{cont,calm}$ and $\lambda_t^1 = p_t\lambda^{calm,cont}$, where $\lambda^{calm,cont}$ is the intensity for jumping from the good to the bad state given that the economy is currently in the good state (and analogously for $\lambda^{cont,calm}$).

For the regime switching models (a) and (b), the dynamics of the risky asset A are then

[6] Notice that it is not meaningful to assume that all jumps lead to a regime shift, since then the state would be observable.

[7] The extension to n assets is provided in Appendix A.

defined by (similarly for B)

$$dS_{A,t}/S_{A,t-} = \mu_A^{p_t} dt + v_A d\widetilde{W}_{A,t} - L_A \sum_{j=2}^{3} dN_{A,t}^{j}, \tag{2}$$

where N_A^2 counts the number of jumps in asset A within the good state. These jumps happen with intensity $\lambda_{A,t}^2 = p_t \lambda_A^{calm}$ where λ_A^{calm} is a constant. Analogously, N_A^3 counts the number of jumps in asset A within the bad state. The intensity is $\lambda_{A,t}^3 = (1-p_t)\lambda_A^{cont}$ with λ_A^{cont} being a constant. Note that the processes N^0 and N^1 do not affect the asset prices directly in the regime switching model since a regime switch is never accompanied by a contemporaneous loss in one of the assets. The loss size L_A and the volatility v_A are the same across states since otherwise agents would be able to infer the state from realized losses or from the realized volatility. Notice that the pure regime switching model follows as a special case if we set the loss size L_A equal to zero. The drift is given by $\mu_A^{p_t} = p_t \mu_A^{calm} + (1-p_t)\mu_A^{cont}$ with constants μ_A^{calm} and μ_A^{cont}. The process $\widetilde{W}_A$ is a Brownian motion that is correlated with the shocks of asset B via $< \widetilde{W}_A, \widetilde{W}_B >_t = \rho \in [-1, 1]$.

Both models can induce additional dependence compared to the standard model by Merton (1969) since the coefficients depend on the state. A transition into the bad state can for instance increase the loss probability. This is a reasonable requirement in our setting and we assume that

$$\lambda_A^{cont} \geq \lambda_A^{calm}. \tag{3}$$

Nevertheless, this connection is indirect in the sense that a transition itself does not go together with a loss. Therefore, we also study a model with contagious jumps.

In framework (c), a contagious jump can occur in the good state ('calm'). It leads to both a loss in one of the assets and to a transition into the bad state ('cont') where additional losses might be more likely. To capture the feature that a loss occurs during a transition into the bad state, the asset dynamics must depend on the counting process N^1 that counts the number of these transitions. Therefore, the asset dynamics now read

$$dS_{A,t}/S_{A,t-} = \mu_A^{p_t} dt + v_A d\widetilde{W}_{A,t} - L_A \sum_{j=1}^{3} dN_{A,t}^{j}. \tag{4}$$

The important difference between dynamics (2) and (4) is that the sum now starts at one since contagious jumps can occur. Notice that, in contrast to N_i^2, the process N_i^1 is even not a Poisson process if we condition on being in the good state. This is because one jump triggers a transition into the bad state. To avoid that the agent is able to back out that a contagious jump has occurred, the corresponding loss size must again be equal to L_A.[8] Notice that in our model a transition back into the calm state is not linked to a jump in the asset prices. Therefore, the asset dynamics do not involve the counting process N^0. For a model with contagious jumps, we introduce the following notation for the jump intensities

$$\lambda^1_{A,t} = p_t \lambda_A^{calm,cont}, \quad \lambda^2_{A,t} = p_t \lambda_A^{calm,clam}, \quad \lambda^3_{A,t} = (1-p_t)\lambda_A^{cont,cont},$$

where $\lambda_A^{calm,cont}$ is now the intensity of a contagious jump (and not only of a regime switch) given that the economy is in the calm state and $\lambda_A^{calm,calm}$ is the intensity of a non-contagious jump. Therefore, the intensity of a jump in the calm state is defined by the sum

$$\lambda_A^{calm,*} = \lambda_A^{calm,cont} + \lambda_A^{calm,clam}.$$

To make sure that both model classes (b) and (c) have the same jump probabilities in both states, we assume that[9]

$$\lambda_A^{calm} = \lambda_A^{calm,*}, \quad \lambda_A^{cont} = \lambda_A^{cont,cont}, \tag{5}$$

where the left-hand sides of the equations are the jump intensities of a regime switching model, whereas the right-hand sides are the intensities of a model with contagious jumps. Moreover, we assume

$$\lambda^{calm,cont} = \lambda_A^{calm,cont} + \lambda_B^{calm,cont} \tag{6}$$

so that a transition from the calm to the contagion state is equally likely in all models (a)-(c). This assumption allows us to isolate the effect of contagious jumps. Finally, since the contagion state is the bad state of the economy, we assume that

$$\lambda_i^{cont,cont} \geq \lambda_i^{calm,*}, \tag{7}$$

i.e. the probability of losses increases in the contagion state, which is analogous to condition (3).

[8]In our model, jump sizes are assumed to be constant. If jump sizes are stochastic, then one can allow for different expected losses in both states without running the risk that the agent can infer the state from realized losses. A model with stochastic jump sizes is for instance analyzed in Kraft and Steffensen (2008).

[9]Notice that there are no 'contagious' jumps from the bad state to the good state, i.e. formally $\lambda_A^{cont,calm} = 0$.

2.2 Unobservable State

The asset price dynamics depend on the current state of the economy. Realistically, however, agents are only able to observe asset prices. In the following, we thus assume that an investor has partial information: Although he knows all model parameters, he cannot observe the state of the economy, but has to infer it from asset prices. Formally, this is captured by two filtrations: The 'large' filtration $\mathcal{F}$ includes all information describing the true data-generating process, while the 'small' filtration $\{\mathcal{G}_t\}_{t\in[0,T]} \subset \{\mathcal{F}_t\}_{t\in[0,T]}$ contains the (partial) information available to investors. The filtration $\{\mathcal{G}_t\}_{t\in[0,T]}$ includes the history of both asset prices, but not the history of the underlying hidden Markov chain. Recall that $p_t \in \{0, 1\}$ is the indicator variable for being in the good state at time t (e.g. $p_t = 0$ if the economy is in the bad state). Therefore, we define $\widehat{p}_t$ as the conditional expectation $\widehat{p}_t = \mathrm{E}[p_t|\mathcal{G}_t]$. In other words, $\widehat{p}_t$ gives the investor's filtered probability of being in the good state at time t.[10]

The investor can perfectly disentangle jumps from diffusions since we assume a continuous-time model.[11] He observes the total number of jumps $\widehat{N}_A$ and $\widehat{N}_B$ defined by[12]

$$\widehat{N}_i = N_i^2 + N_i^3, \quad i = A, B,$$

in the regime switching model and

$$\widehat{N}_i = N_i^1 + N_i^2 + N_i^3, \quad i = A, B,$$

in the contagion model. In the contagion model, the agent is however not able to distinguish between the three different kinds of jumps on the right hand side (jumps within a state vs. jumps upon transition). Furthermore, he cannot observe jumps back from the contagion state to the calm state since these jumps do not have an impact on the asset prices.

2.3 Filtering the State of the Economy

The next step is to determine the probability dynamics of being in the calm state. Since we wish to study the differences between regime switching models and our model with

[10] Note that $\widehat{p}_t = \mathrm{E}[p_t|\mathcal{G}_t]$ is the estimate for p_t that minimizes the mean-square distance between p_t and all square-integrable and $\mathcal{G}_t$-measurable random variables. See, e.g., Brémaud (1981).

[11] This is, at least asymptotically, even possible in discrete-time models, see e.g. Ait-Sahalia (2004) or Johannes, Polson, and Stroud (2009).

[12] We will stick to this notational convention throughout the remainder of the paper. Variables with a 'hat' denote subjective numbers that the investor estimates from his observations. Variables without a 'hat' represent the true numbers in the economy.

contagious jumps, we derive filter equations for both model classes. In portfolio problems with unobservable states, these dynamics serve as an additional state variable.[13] Furthermore, we are interested in the filtered jump intensities of asset prices. For a regime switching model with jumps, they are defined by

$$\widehat{\lambda}_i^{rs} = \widehat{p}^{rs}\lambda_i^{calm} + (1-\widehat{p}^{rs})\lambda_i^{cont}, \tag{8}$$

whereas for a model with contagious jumps we have

$$\widehat{\lambda}_i = \widehat{p}\left(\lambda_i^{calm,calm} + \lambda_i^{calm,cont}\right) + (1-\widehat{p})\lambda_i^{cont,cont}. \tag{9}$$

The following proposition summarizes our results that are formulated for n assets.[14]

Proposition 1 (Filtered Probability) *(i) In a regime switching economy with n assets, the filtered probability of being in the calm state is given by*[15]

$$\begin{aligned} d\widehat{p}_t^{rs} &= \left((1-\widehat{p}_t^{rs})\lambda^{cont,calm} + \widehat{p}_t^{rs}\left\{\sum_{i=1}^n \widehat{\lambda}_i^{rs} - \sum_{i=1}^n \lambda_i^{calm}\right\} - \widehat{p}_t^{rs}\lambda^{calm,cont}\right)dt \qquad (10)\\ &\quad + \widehat{p}_t^{rs}(1-\widehat{p}_t^{rs})(\mu^{calm}-\mu^{cont})\Sigma^{-T}d\widehat{W}_t + \sum_{i=1}^n \left(\frac{\widehat{p}_{t-}^{rs}\lambda_i^{calm}}{\widehat{\lambda}_i^{rs}} - \widehat{p}_{t-}^{rs}\right)d\widehat{N}_{i,t}. \end{aligned}$$

(ii) In a model with contagious jumps and n assets, the filtered probability of being in the calm state is given by

$$\begin{aligned} d\widehat{p}_t &= \left((1-\widehat{p}_t)\lambda^{cont,calm} + \widehat{p}_t\left\{\sum_{i=1}^n \widehat{\lambda}_i - \sum_{i=1}^n \lambda_i^{calm,*}\right\}\right)dt \qquad (11)\\ &\quad + \widehat{p}_t(1-\widehat{p}_t)(\mu^{calm}-\mu^{cont})\Sigma^{-T}d\widehat{W}_t + \sum_{i=1}^n \left(\frac{\widehat{p}_{t-}\lambda_i^{calm,calm}}{\widehat{\lambda}_i} - \widehat{p}_{t-}\right)d\widehat{N}_{i,t}. \end{aligned}$$

Remark. In one of our applications, we consider the case where the investor disregards diffusive information. The corresponding (suboptimal) filter follows if the diffusion term in (11) are disregarded.

The two filter equations have identical diffusive terms, which are known from Honda (2003). The equations however differ in two respects: Firstly, the drift terms differ (compare the dt-terms in (10) and (11)). In the regime switching model, there is an additional

[13] See, e.g., Detemple (1986) and Dothan and Feldman (1986).

[14] The proofs can be found in Appendix A.

[15] A special case for one asset and without filtering from different drifts appears in Bäuerle and Rieder (2007). Their model also does not involve contagious jumps.

term, $-\widehat{p}_t^{rs}\lambda^{calm,cont}$, that is always negative. The rest of the drift (and also the drift in the model with contagious jumps) is always positive since, by definitions (8) and (9), the differences in the curly brackets are positive. Therefore, in the model with contagious jumps the drift is always positive and $\widehat{p}$ is drifting upwards towards one as long as no jump occurs, whereas, in the regime switching model, the drift becomes negative if $\widehat{p}^{rs}$ is close to one. In the latter case, the mean reversion level of $\widehat{p}_t^{rs}$ is thus between 0 and 1. Secondly, there is a difference in the jump sizes, which can be interpreted as updates of the filters if asset price jumps occur. In the model with contagious jumps, the jump size is larger because a jump can be contagious. The occurrence of a jump reveals more negative information than in a regime switching model.

2.4 Relation to Self-exciting Processes

Now, we determine the implied filtered jump intensity of an asset and compare it to the self-exciting model (1). For simplicity, this section focuses on the one asset case.[16] By (8) and (9), the dynamics of the implied filtered jump intensity are given by

$$d\widehat{\lambda}_i^{rs} = (\lambda_i^{calm} - \lambda_i^{cont})d\widehat{p}^{rs}, \qquad d\widehat{\lambda}_i = (\lambda_i^{calm,*} - \lambda_i^{cont,cont})d\widehat{p}.$$

Using one-dimensional versions of (10) and (11) with $n = 1$ and $i = A$ yields the following results.

Proposition 2 (Filtered Jump Intensities) *(i) In a regime switching economy with one asset, the implied filtered jump intensity is given by*

$$d\widehat{\lambda}_{A,t}^{rs} = \varsigma_{A,t}^{rs}dt + \zeta_{A,t}^{rs}d\widehat{W}_{A,t} + \ell_{A,t}^{rs}d\widehat{N}_{A,t},$$

where

$$\begin{aligned}
\varsigma_{A,t}^{rs} &= -(\widehat{\lambda}_{A,t}^{rs} - \lambda_A^{calm})\lambda^{cont,calm} - (\widehat{\lambda}_{A,t}^{rs} - \lambda_A^{calm})(\lambda_A^{cont} - \widehat{\lambda}_{A,t}^{rs}) + (\lambda_A^{cont} - \widehat{\lambda}_{A,t}^{rs})\lambda^{calm,cont} \\
\zeta_{A,t}^{rs} &= \frac{(\lambda_A^{cont} - \widehat{\lambda}_{A,t}^{rs})(\widehat{\lambda}_{A,t}^{rs} - \lambda_A^{calm})}{\lambda_A^{cont} - \lambda_A^{calm}} \cdot \frac{\mu_A^{cont} - \mu_A^{calm}}{v_A} \\
\ell_{A,t}^{rs} &= (\lambda_A^{cont} - \widehat{\lambda}_{A,t}^{rs}) \cdot \frac{\widehat{\lambda}_{A,t}^{rs} - \lambda_A^{calm}}{\widehat{\lambda}_{A,t}^{rs}}.
\end{aligned}$$

(ii) In a model with contagious jumps and one asset, the implied filtered jump intensity is given by

$$d\widehat{\lambda}_{A,t} = \varsigma_{A,t}dt + \zeta_{A,t}d\widehat{W}_{A,t} + \ell_{A,t}d\widehat{N}_{A,t},$$

[16] The generalization to a multi-asset case is possible and would lead to mutually exciting point processes as discussed by Ait-Sahalia, Cacho-Diaz, and Laeven (2012). This is a multi-dimensional version of (1).

where

$$\begin{aligned}
\varsigma_{A,t} &= -(\widehat{\lambda}_{A,t} - \lambda_A^{calm,*})\lambda^{cont,calm} - (\widehat{\lambda}_{A,t} - \lambda_A^{calm,*})(\lambda_A^{cont,cont} - \widehat{\lambda}_{A,t}) \\
\zeta_{A,t} &= \frac{(\lambda_A^{cont,cont} - \widehat{\lambda}_{A,t})(\widehat{\lambda}_{A,t} - \lambda_A^{calm,*})}{\lambda_A^{cont,cont} - \lambda_A^{calm,*}} \cdot \frac{\mu_A^{cont} - \mu_A^{calm}}{\upsilon_A} \\
\ell_{A,t} &= (\lambda_A^{cont,cont} - \widehat{\lambda}_{A,t}) \cdot \frac{\widehat{\lambda}_{A,t} - \lambda_A^{calm,calm}}{\widehat{\lambda}_{A,t}}.
\end{aligned}$$

First, the filtered probabilities are differently updated as a response to asset price jumps: If there are contagious jumps ($\lambda_A^{calm,cont} > 0$), then $\lambda_A^{calm} > \lambda_A^{calm,calm}$ (see condition (5)). Therefore, everything else equal, we obtain $\ell_{A,t} > \ell_{A,t}^{rs}$, i.e. the jump of the filtered probability is larger in the model with contagious jumps. This is because asset price jumps are more informative in this model.

Second, in the regime switching model, the drift $\varsigma_{A,t}^{rs}$ can be decomposed into three terms: The first term always pulls the intensity downwards because of the possibility to switch from the bad into the good state, which cannot be observed. The second term is always negative and is driven by the fact that the jump intensities in both states differ. When no jumps occur, the investor can learn from this information, since on average there are less jumps in the calm state. The third term, which always drives up the intensity, is present since in the regime switching model changes from the calm to the contagion state are unobservable. This third term does not show up in the dynamics of the model with contagious jumps where the drift can be rewritten as

$$\kappa_t(\lambda_A^{calm,*} - \widehat{\lambda}_{A,t})$$

with state dependent mean reversion speed $\kappa_t = \lambda^{cont,calm} + \lambda^{cont,cont} - \widehat{\lambda}_{A,t} > 0$. In the model with contagious jumps, $\lambda_A^{calm,*}$ can be interpreted in a similar way as θ in (1). In particular, $\lambda_A^{calm,*}$ is a lower bound for $\widehat{\lambda}_{A,t}$. In a regime switching economy, the additional third term, $(\lambda^{cont} - \widehat{\lambda}_{A,t}^{rs})\lambda^{calm,cont}$, changes this property. More precisely, the drift $\varsigma_{A,t}^{rs}$ is a quadratic function of $\widehat{\lambda}^{rs}$. For $\widehat{\lambda}^{rs} = \lambda^{calm}$ the drift is positive, and for $\widehat{\lambda}^{rs} = \lambda^{cont}$ it is negative. Consequently, there must be a root between these two points. The process $\widehat{\lambda}^{rs}$ is thus fluctuating around a level λ^* that is between λ^{calm} and λ^{cont}. Therefore, $\widehat{\lambda}^{rs}$ can have a positive or negative drift, which distinguishes $\widehat{\lambda}^{rs}$ from a self-exciting process and which is in sharp contrast to the filtered intensity $\widehat{\lambda}$ in a model with contagious jumps.

To understand this point, consider the general case with diffusion, $\zeta_{A,t}^{rs} \neq 0$. Then $\widehat{\lambda}^{rs}$ can fall below λ^*. Although the drift of $\widehat{\lambda}^{rs}$ vanishes at λ^*, diffusive shocks can push $\widehat{\lambda}^{rs}$ below λ^*. By the definition of λ^*, this leads to a positive drift. Notice however that in the special case where there is no diffusion in the regime switching model and the process $\widehat{\lambda}^{rs}$ starts at a level above λ^*, the filtered probability $\widehat{\lambda}^{rs}$ can never fall below λ^*. This is because the jump size is always positive and the drift becomes zero near λ^*.

3 Optimal Portfolio Choice

This section is concerned with studying the effects of the different model specifications on asset allocation. In particular, we analyze the impact of contagious jumps. Notice that the portfolio problem can be solved in two steps. In the first step, the investor solves a filtering problem, i.e. he estimates the current value of the state variable. Secondly, he decides on his optimal portfolio conditional on the just estimated state variable and its dynamics.[17]

3.1 Optimization Problem

We consider an investor with CRRA utility $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ where $\gamma > 0$ denotes his relative risk aversion. The planning horizon is denoted by T. The investor maximizes expected utility from terminal wealth X_T. His indirect utility at time t depends on his wealth X_t and the filtered probability of being in the good state, $\widehat{p}_t$, which, depending on the applied filter, follows the dynamics given in Section 2.3. It is defined as

$$G(t, x, \widehat{p}_t) = \max_{\pi \in \mathcal{A}(t,\widehat{p}_t)} \mathrm{E}^{t,x}\big[u(X_T)|\mathcal{G}_t\big]$$

where $\mathcal{A}(t, \widehat{p}_t)$ denotes the set of all admissible trading strategies and $\pi = (\pi_A, \pi_B)$. Due to event risk, the investor faces an incomplete market. In order to choose optimal exposures to the different sources of risk (diffusion and jumps), he can adjust the weights π_A and π_B of the two risky assets in his portfolio. His budget constraint reads

$$\frac{dX_t}{X_{t-}} = \pi_A(t, \widehat{p}_t)\frac{dS_{A,t}}{S_{A,t-}} + \pi_B(t, \widehat{p}_t)\frac{dS_{B,t}}{S_{B,t-}} + \big[1 - \pi_A(t, \widehat{p}_t) - \pi_B(t, \widehat{p}_t)\big]r dt. \tag{12}$$

In the remainder of this section, we solve for the indirect utility functions and the optimal portfolio weights in three different cases: First, we consider our model with contagious jumps. Second, we derive the solution in this model if the agent disregards information stemming from diffusive shocks. Third, we study a regime-switching model. In all three cases, we conjecture that the indirect utility is given by

$$G(t, x, \widehat{p}) = \frac{x^{1-\gamma}}{1-\gamma} f(t, \widehat{p}), \tag{13}$$

[17] See, e.g., Detemple (1986), Dothan and Feldman (1986), and Bäuerle and Rieder (2007).

where x denotes current wealth and $\widehat{p}$ the filtered probability. The corresponding function f is part of the solution and must be determined either explicitly or numerically.

3.2 Portfolio Choice with Contagious Jumps

In this model, the agent uses the filter (11). His budget constraint is given by (12). The indirect utility function $G(t, x, \widehat{p})$ solves the Bellman equation[18]

$$
\begin{aligned}
\max_{\pi_A,\pi_B} \Big\{ & G_t + G_x[r + \pi_A(\widehat{\mu}_A - r) + \pi_B(\widehat{\mu}_B - r)] \\
& + G_p\Big[(1-\widehat{p}_t)\lambda^{cont,calm} - \widehat{p}_t(\lambda_A^{calm,cont} + \lambda_B^{calm,cont}) \\
& \qquad - \widehat{p}_t(\lambda_A^{calm,calm} - \widehat{\lambda}_A) - \widehat{p}_t(\lambda_B^{calm,calm} - \widehat{\lambda}_B)\Big] \\
& + 0.5G_{xx}\left[v_A^2\pi_A^2 + 2\rho v_A v_B \pi_A \pi_B + v_B^2\pi_B^2\right] \\
& + 0.5G_{pp}\frac{\widehat{p}^2(1-\widehat{p})^2}{1-\rho^2}\left[\frac{(\mu_A^{calm}-\mu_A^{cont})^2}{v_A^2} - 2\rho\frac{(\mu_A^{calm}-\mu_A^{cont})(\mu_B^{calm}-\mu_B^{cont})}{v_A v_B} + \frac{(\mu_B^{calm}-\mu_B^{cont})^2}{v_B^2}\right] \\
& + G_{px}\Big[(1-\gamma)\widehat{p}(1-\widehat{p})\big(\pi_A(\mu_A^{calm}-\mu_A^{cont}) + \pi_B(\mu_B^{calm}-\mu_B^{cont})\big) \\
& \qquad + (1-\widehat{p})\lambda^{cont,calm} - \widehat{p}(\lambda_A^{calm,cont} + \lambda_B^{calm,cont}) + \widehat{p}\big(\widehat{\lambda}_A + \widehat{\lambda}_B - \lambda_A^{calm,calm} - \lambda_B^{calm,calm}\big)\Big] \\
& + \big(G^{A,+} - G\big)\widehat{\lambda}_A + \big(G^{B,+} - G\big)\widehat{\lambda}_B\Big\} \;=\; 0,
\end{aligned}
$$

where subscripts denote partial derivatives. The notation $G^{i,+}$ refers to the function G immediately after a jump in asset $i \in \{A, B\}$. Substituting the conjecture (13) into the Bellman equation yields a system of equations for f and the optimal demands, π_A and π_B, that can be solved numerically. The following proposition summarizes our results.[19]

Proposition 3 (Solution with Contagious Jumps) *In a model with contagious jumps, the optimal portfolio weights satisfy the first-order conditions*

$$
\begin{aligned}
f \cdot \left[\widehat{\mu}_A - r - \gamma(\pi_A v_A^2 + \rho v_A v_B \pi_B)\right] + f_p \cdot \widehat{p}(1-\widehat{p})(\mu_A^{calm} - \mu_A^{cont}) & \\
-f\left(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}\right) \cdot L_A(1-\pi_A L_A)^{-\gamma}\widehat{\lambda}_A &= 0 \\
f \cdot \left[\widehat{\mu}_B - r - \gamma(\pi_B v_B^2 + \rho v_A v_B \pi_A)\right] + f_p \cdot \widehat{p}(1-\widehat{p})(\mu_B^{calm} - \mu_B^{cont}) & \\
-f\left(t, \frac{\lambda_B^{calm,calm}}{\widehat{\lambda}_B}\widehat{p}\right) \cdot L_B(1-\pi_B L_B)^{-\gamma}\widehat{\lambda}_B &= 0,
\end{aligned}
$$

[18]Qualitatively, the Bellman equation is similar to the equations by Kraft and Steffensen (2008) and Kraft and Steffensen (2009) where the interested reader can find more information on verification results.

[19]The proof is given in Appendix B.

where f solves

$$
\begin{aligned}
& f \cdot \Big[(1-\gamma)r + (1-\gamma)\pi_A(\widehat{\mu}_A - r) + (1-\gamma)\pi_B(\widehat{\mu}_B - r) \\
& \quad -0.5\gamma(1-\gamma)\big(v_A^2\pi_A^2 + 2\rho v_A v_B \pi_A \pi_B + v_B^2\pi_B^2\big) - \widehat{\lambda}_A - \widehat{\lambda}_B\Big] \\
& + f_p \cdot \Big[(1-\gamma)\widehat{p}(1-\widehat{p})\big(\pi_A(\mu_A^{calm} - \mu_A^{cont}) + \pi_B(\mu_B^{calm} - \mu_B^{cont})\big) \\
& \quad +(1-\widehat{p})\lambda^{cont,calm} - \widehat{p}(\lambda_A^{calm,cont} + \lambda_B^{calm,cont}) + \widehat{p}\big(\widehat{\lambda}_A + \widehat{\lambda}_B - \lambda_A^{calm,calm} - \lambda_B^{calm,calm}\big)\Big] \\
& + f_{pp} \cdot \frac{0.5\widehat{p}^2(1-\widehat{p})^2}{1-\rho^2}\left[\frac{(\mu_A^{calm} - \mu_A^{cont})^2}{v_A^2} - 2\rho\frac{(\mu_A^{calm} - \mu_A^{cont})(\mu_B^{calm} - \mu_B^{cont})}{v_A v_B} + \frac{(\mu_B^{calm} - \mu_B^{cont})^2}{v_B^2}\right] \\
& + f\left(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}\right) \cdot (1-\pi_A L_A)^{1-\gamma}\widehat{\lambda}_A + f\left(t, \frac{\lambda_B^{calm,calm}}{\widehat{\lambda}_B}\widehat{p}\right) \cdot (1-\pi_B L_B)^{1-\gamma}\widehat{\lambda}_B \quad + f_t = 0
\end{aligned}
\tag{14}
$$

with boundary conditions $f(T,\cdot) = 1$ and $f_p(T,\cdot) = 0$.

As usually in incomplete market problems with jumps, the first-order conditions and the Bellman equation (14) can only be solved simultaneously.

The indirect utility function and the optimal portfolio weights π_i depend on the state variable $\widehat{p}$. Since $\widehat{p}$ evolves stochastically following a jump-diffusion process, the optimal portfolio weights are also stochastic. Besides, the demands are monotonic functions of $\widehat{p}$. Consequently, as long as no jump is observed, they continuously revert back to the optimal portfolio for $\widehat{p} = 1$ where the investor is sure to be in the calm state. If a jump occurs, the portfolio weights are reduced by a discrete amount towards the optimal portfolio for $\widehat{p} = 0$.

To interpret the optimal portfolio strategy, assume for simplicity that the correlation ρ between the diffusion components is zero. The first-order condition for asset A can then be rewritten as

$$
\pi_A = \frac{\widehat{\mu}_A - r}{\gamma v_A^2} + \widehat{p}(1-\widehat{p})\frac{\mu_A^{calm} - \mu_A^{cont}}{\gamma v_A^2}\frac{f_p}{f} - \frac{f\big(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}\big)}{f}\frac{L_A(1-\pi_A L_A)^{-\gamma}\widehat{\lambda}_A}{\gamma v_A^2} \tag{15}
$$

The optimal portfolio strategy consists of three parts:[20] The first term is the myopic demand that depends on the filtered probability of being in the calm state. This term is a weighted average of the optimal demands in the two states if the state was known with certainty:

$$
\frac{\widehat{\mu}_A - r}{\gamma v_A^2} = \widehat{p}\,\frac{\mu_A^{calm} - r}{\gamma v_A^2} + (1-\widehat{p})\frac{\mu_A^{cont} - r}{\gamma v_A^2}. \tag{16}
$$

[20]In the special case of a pure regime switching model ($L_A = 0$), we recover the result of Honda (2003).

The second term captures hedging motives stemming from the continuous updating of $\widehat{p}$ due to diffusion. The hedge term is large if there is a lot of uncertainty about the state ($\widehat{p} \approx 0.5$), if the states are heterogenous with respect to the drifts ($|\mu_A^{calm} - \mu_A^{cont}|$ large), if the signal is not too noisy (v_A small), and if the indirect utility is sensitive to changes in the filtered probability ($|f_p|/f$ large). The sign of the term depends on which of the two states is more attractive. This is determined by the risk premia in the states. In fact, the derivative f_p can be both positive or negative.

The third term adjusts the portfolio strategy with respect to possible crashes in the asset. Since contagious jumps involve two aspects (regime switch and loss), this term consists of two parts. First, there is a term that accounts for the loss and that is also present in a model with event risk, but no state variable (see, e.g., Liu, Longstaff, and Pan (2003)):

$$-\frac{L_A(1-\pi_A L_A)^{-\gamma}\widehat{\lambda}_A}{\gamma v_A^2}. \tag{17}$$

This term is negative and thus adjusts the demand downwards. In our model, the demand additionally involves the ratio $f\big(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}\big)/f$, which amplifies or dampens the adjustment term (17) depending on which state is more attractive. For instance, if the expected returns in the contagion state are smaller than in the calm state, then $f_p < 0$ and this ratio is greater than one, which puts more weight on (17) so that the demand arising from jump risk becomes more negative. The ratio is present due to the optimal updating of the filtered probability. If the asset price jumps, then the probability of being in the calm state, $\widehat{p}$, decreases to $\widehat{p}\lambda_A^{calm,calm}/\widehat{\lambda}_A$ since $\lambda_A^{calm,calm} < \widehat{\lambda}_A$.

Finally, we wish to remark that the second and third term (the hedging terms for diffusive and jump risk) partly cancel each other out. This is for the following reasons: The third term is always negative. The sign of the second term depends on the signs of f_p and $\mu_A^{calm} - \mu_A^{cont}$. Since, by definition, the contagion regime is a bad state, it is likely that $\mu_A^{calm} - \mu_A^{cont}$ is positive. If $\mu_A^{calm} - \mu_A^{cont}$ is positive, the contagion state is perceived worse than the calm state so that f_p is negative. In this case, $f\big(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}\big)/f$ is larger than one. Therefore, the second term is positive if the third term is very negative and vice versa.

3.3 Portfolio Choice with Pure Jump Filter

In our numerical analysis, we also study the case where the investor ignores diffusive information and relies on the information from jumps only. In a model with contagious jumps, one might argue that this information might be 'sufficient' to find an 'almost optimal' portfolio strategy. In fact, our numerical results show that the utility loss from ignoring diffusive information is negligible if there are contagious jumps. The indirect utility is still of the form (13).

Proposition 4 (Solution with Pure Jump Filter) *If the investor uses the pure jump filter to estimate the current state of the economy, the optimal portfolio weights satisfy the first-order conditions*

$$f \cdot \left[\widehat{\mu}_A - r - \gamma(\pi_A v_A^2 + \rho v_A v_B \pi_B)\right] - f\left(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}^{pjf}\right) \cdot L_A(1-\pi_A L_A)^{-\gamma}\widehat{\lambda}_A = 0,$$

$$f \cdot \left[\widehat{\mu}_B - r - \gamma(\pi_B v_B^2 + \rho v_A v_B \pi_A)\right] - f\left(t, \frac{\lambda_B^{calm,calm}}{\widehat{\lambda}_B}\widehat{p}^{pjf}\right) \cdot L_B(1-\pi_B L_B)^{-\gamma}\widehat{\lambda}_B = 0,$$

where

$$\begin{aligned}
&f \cdot \Big[(1-\gamma)r + (1-\gamma)\pi_A(\widehat{\mu}_A - r) + (1-\gamma)\pi_B(\widehat{\mu}_B - r) \\
&\quad -0.5\gamma(1-\gamma)\big(v_A^2\pi_A^2 + 2\rho v_A v_B \pi_A \pi_B + v_B^2 \pi_B^2\big) - \widehat{\lambda}_A - \widehat{\lambda}_B\Big] \\
&+f_p \cdot \Big[(1-\widehat{p}^{pjf})\lambda^{cont,calm} - \widehat{p}^{pjf}(\lambda_A^{calm,cont} + \lambda_B^{calm,cont}) + \widehat{p}^{pjf}\big(\widehat{\lambda}_A + \widehat{\lambda}_B - \lambda_A^{calm,calm} - \lambda_B^{calm,calm}\big)\Big] \\
&+f\left(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}^{pjf}\right) \cdot (1-\pi_A L_A)^{1-\gamma}\widehat{\lambda}_A \\
&+f\left(t, \frac{\lambda_B^{calm,calm}}{\widehat{\lambda}_B}\widehat{p}^{pjf}\right) \cdot (1-\pi_B L_B)^{1-\gamma}\widehat{\lambda}_B \quad + f_t = 0
\end{aligned}$$

with boundary conditions $f(T,\cdot) = 1$ *and* $f_p(T,\cdot) = 0$. *The subjective drift rate and jump intensity of asset* $i \in \{A, B\}$ *are defined as*

$$\begin{aligned}
\widehat{\mu}_i &= \widehat{p}^{pjf}\mu_i^{calm} + (1-\widehat{p}^{pjf})\mu_i^{cont} \\
\widehat{\lambda}_i &= \widehat{p}^{pjf}\left(\lambda_i^{calm,calm} + \lambda_i^{calm,cont}\right) + (1-\widehat{p}^{pjf})\lambda_i^{cont,cont}.
\end{aligned}$$

3.4 Portfolio Choice in a Regime Switching Model

Finally, we consider a regime switching model with jumps. The solutions can be found analogously to the results in Honda (2003) and Bäuerle and Rieder (2007).

Proposition 5 (Solution in a Regime Switching Model) *In a regime switching model* *with jumps, the optimal portfolio weights satisfy the first-order conditions*

$$
\begin{aligned}
f \cdot \left[\widehat{\mu}_A - r - \gamma(\pi_A v_A^2 + \rho v_A v_B \pi_B)\right] + f_p \cdot \widehat{p}^{rs}(1-\widehat{p}^{rs})(\mu_A^{calm} - \mu_A^{cont}) & \\
- f\left(t, \frac{\lambda_A^{calm}}{\widehat{\lambda}_A^{rs}}\widehat{p}^{rs}\right) \cdot L_A(1-\pi_A L_A)^{-\gamma}\widehat{\lambda}_A^{rs} &= 0 \\
f \cdot \left[\widehat{\mu}_B - r - \gamma(\pi_B v_B^2 + \rho v_A v_B \pi_A)\right] + f_p \cdot \widehat{p}^{rs}(1-\widehat{p}^{rs})(\mu_B^{calm} - \mu_B^{cont}) & \\
- f\left(t, \frac{\lambda_B^{calm}}{\widehat{\lambda}_B^{rs}}\widehat{p}^{rs}\right) \cdot L_B(1-\pi_B L_B)^{-\gamma}\widehat{\lambda}_B^{rs} &= 0
\end{aligned}
$$

where f solves

$$
\begin{aligned}
& f \cdot \Big[(1-\gamma)r + (1-\gamma)\pi_A(\widehat{\mu}_A - r) + (1-\gamma)\pi_B(\widehat{\mu}_B - r) \\
& \quad -0.5\gamma(1-\gamma)\big(v_A^2\pi_A^2 + 2\rho v_A v_B \pi_A \pi_B + v_B^2 \pi_B^2\big) - \widehat{\lambda}_A^{rs} - \widehat{\lambda}_B^{rs}\Big] \\
& + f_p \cdot \Big[(1-\gamma)\widehat{p}^{rs}(1-\widehat{p}^{rs})\big(\pi_A(\mu_A^{calm} - \mu_A^{cont}) + \pi_B(\mu_B^{calm} - \mu_B^{cont})\big) \\
& \quad + (1-\widehat{p}^{rs})\lambda^{cont,calm} - \widehat{p}^{rs}\lambda^{calm,cont} + \widehat{p}^{rs}(\widehat{\lambda}_A^{rs} + \widehat{\lambda}_B^{rs} - \lambda_A^{calm} - \lambda_B^{calm})\Big] \\
& + f_{pp} \cdot \frac{0.5(\widehat{p}^{rs})^2(1-\widehat{p}^{rs})^2}{1-\rho^2}\left[\frac{(\mu_A^{calm} - \mu_A^{cont})^2}{v_A^2} - 2\rho\frac{(\mu_A^{calm} - \mu_A^{cont})(\mu_B^{calm} - \mu_B^{cont})}{v_A v_B} + \frac{(\mu_B^{calm} - \mu_B^{cont})^2}{v_B^2}\right] \\
& + f\left(t, \frac{\lambda_A^{calm}}{\widehat{\lambda}_A^{rs}}\widehat{p}^{rs}\right) \cdot (1-\pi_A L_A)^{1-\gamma}\widehat{\lambda}_A^{rs} f\left(t, \frac{\lambda_B^{calm}}{\widehat{\lambda}_B^{rs}}\widehat{p}^{rs}\right) \cdot (1-\pi_B L_B)^{1-\gamma}\widehat{\lambda}_B^{rs} \quad + f_t = 0
\end{aligned}
$$

with boundary conditions $f(T,\cdot) = 1$ and $f_p(T,\cdot) = 0$.

4 Numerical Results

4.1 Parametrization and Calibration

We consider a CRRA investor with a relative risk aversion of $\gamma = 3$ and a planning horizon of 20 years. The riskless interest rate is set to $r = 0.01$. The risky assets are assumed to follow identical stochastic processes. Furthermore, we assume that only the jump intensities and the drift rates differ between the calm and the contagion state, while the diffusion parameters and the loss sizes do not depend on the current state as already explained above. We choose representative parameters for our model that are roughly in line with Eraker, Johannes, and Polson (2003) who estimate the parameters of a jump-diffusion model under the true physical measure from S&P500 and Nasdaq 100 index returns.

The diffusion volatility σ is set to 0.15 and the Brownian motions have a local correlation of $\rho = 0.3$. The constant jump size is assumed to be -5%, i.e. the loss size L_i equals

0.05. The difference between the jump intensities in the calm and the contagion state is captured by the multiple $\xi_i \geq 1$:

$$\lambda_i^{cont,cont} = \xi_i \lambda_i^{calm,*}, \quad i \in \{A, B\},$$

where we set $\xi_i = 5$ and $\lambda_i^{calm,*} = 0.5$. The conditional probability that a loss in one of the assets triggers contagion is given by α_i:

$$\lambda_i^{calm,cont} = \alpha_i \lambda_i^{calm,*}, \quad i \in \{A, B\},$$

where we set $\alpha_i = 0.25$. This determines all intensities and their values are reported in Table 1. The intensity for transitions back to the calm state, $\lambda^{cont,calm}$, is set to 1 so that a contagion period lasts on average one year. The drift μ_i is 0.09 in the calm state and 0.14 in the contagion state. This implies expected annual returns of the risky assets of 0.065 in the calm state and 0.015 in the contagion state, i.e. the higher jump intensities in the contagion state do not only increase the conditional variance of stock returns, but also affect the expected returns. The first column of Table 1 gives an overview over this parametrization.

As a robustness check, we introduce four other parametrizations where we vary one particular parameter while leaving all the other parameters unchanged. In these cases, we change α_i from 0.25 to 0.5, $\lambda^{cont,calm}$ from 1 to 2, γ from 3 to 10 and ρ from 0.3 to 0.6, respectively.

4.2 Filter Dynamics

Figure 1 depicts typical sample paths of both assets. The economy is in the calm state for the first five years and then jumps into the contagion state for one year where the jump probabilities are higher. Finally, the calm state is reached again. The transition into the contagion state is triggered by a loss in asset A. We thus assume that there is a contagious jump after five years. Consequently, we allow for contagious jumps in the data generating model. This assumption is supported by recent empirical evidence (see Section 1). Notice however that in the first five years the price paths would be identical in 'both worlds', with and without contagious jumps. Therefore, this period is well suited to compare the behavior of both filters, independent of an assumption about whether contagious jumps exist.

The corresponding filter dynamics are shown in the upper panel of Figure 2. The red line corresponds to the filter for a model with contagious jumps (contagious filter), whereas the black line depicts the realized path of the filter for a model with regime switching (regime switching filter). There are two clear differences between the filters: First, the jumps of the contagious filter are larger. This is because jumps in a contagion model reveal more information than in the regime switching model without contagious jumps. Under the given parametrization, on average every fourth jump in the calm state is contagious so that the update of the filter after the observation of a jump is larger in the model with contagious jumps.

Second, the regime switching filter is noisier. The differences result from the fact that, in contrast to the regime switching filter, the contagious filter induces a self-exciting model. As already pointed out in Section 2.3, the mean reversion levels of the two models are located at different places: For the contagious filter the mean-reversion level of the estimated jump intensity corresponds to the minimal jump intensity (or probability), whereas for the regime switching filter the level is higher. This is because the regime switching filter assumes that the economy can always silently slip into the bad state. Therefore, the reaction to jumps is more pronounced for the contagious filter and its diffusion is shut down if the filter reaches its mean reversion level. On the contrary, the volatility of the regime switching filter is still positive around its mean reversion level and thus the filter is noisier.

4.3 Optimal Portfolios

The filter dynamics described in the previous section have a direct effect on the optimal portfolio weights, which are depicted in the lower panel of Figure 2. Since both assets have identical parameters, the portfolio weights for asset A and asset B are equal. It can be seen that the portfolio weights vary significantly with the filters, since there is a monotonous dependence between the weights and the filtered probabilities. In particular, the weight dynamics are noisier for the regime switching filter and the weight updates after a jump are more pronounced for the contagious filter. Notice that in our numerical example the optimal portfolio weights vary between about 60%, if the investor is sure to be in the calm state, and 5%, if the investor is sure to be in the contagion state.

Figure 2 shows that the investor underreacts to jumps that induce contagion and overreacts to ordinary jumps. If a jump in one of the assets triggers contagion, a fully informed investor should switch to the optimal portfolio of the contagion state in one single step. However, the reaction of an investor with incomplete information is too small, and it takes several subsequent jumps for the investor to gradually adjust his portfolio towards the portfolio that is optimal in the contagion state (contagion portfolio). If, on the other hand, a non-contagious jump occurs, then the investor overreacts to this event by adjusting the weights towards the contagion portfolio while a fully informed investor would have kept the weights constant. If no subsequent jumps are observed, the partially informed investor will then continuously readjust his portfolio back to the portfolio that is optimal when the filtered probability is at the mean reversion level. For the contagious filter the mean-reversion level of the estimated probability is one, whereas for the regime switching filter the level is higher. Interestingly, these over- and underreaction patterns occur for both filters (contagious and regime switching), but the overreaction is less pronounced for the regime switching filter.

It is important to remark that for a *given* filtered probability the two weights are not very different per se. This can be seen in Figure 3 where the solid red line depicts the optimal weights with contagious jumps, whereas the dashed-dotted blue line depicts the weights for a regime switching model as a function of the filtered probability $\widehat{p}$. Since the optimal weights depend on $\widehat{p}$ almost linearly, most of the deviations between the two strategies (see lower panel of Figure 2) come from the difference in the filtered probabilities that are shown in the upper panel of Figure 2. For instance, the update of the contagious filter upon a loss is more pronounced and thus this jump in the filtered probability has a larger effect on the portfolio weight, which can be seen in Figure 4. The portfolio update is particularly large for $\widehat{p}$ around 0.6 where the uncertainty is very large. Moreover, the portfolio update in the regime switching model is 0 if $\widehat{p}$ equals 1. Therefore, if the investor was sure to be in the calm state, the occurrence of a jump would not affect his opinion about the state of the economy.

4.4 Misspecification

In the previous subsections, we have not taken a clear stance on whether there are contagious jumps in the data generating model, but have analyzed the qualitative aspects of the two frameworks. Now, we quantify the utility losses if there are contagious jumps, but the agent ignores them and filters using the regime switching filter. For comparison, we also consider situations where the agent disregards diffusive information or does not filter at all. For this exercise, the solution to the model with contagious jumps serves as benchmark and we express all utility losses relative to this case. More precisely, we calculate the percentage decreases δ in initial wealth that are necessary to reduce the expected utility of the optimal strategy to the expected utilities of some of the other strategies, i.e. $G_{opt}(x(1-\delta)) = G_{subopt}(x)$ where G_{opt} is the indirect utility function in the model with contagious jumps and G_{subopt} is the expected utility of a suboptimal strategy.[21] The form

[21] We have omitted the dependence on time and the filtered probability.

(13) of the indirect utility functions yields[22]

$$\delta = 1 - \left(\frac{f_{subopt}}{f_{opt}} \right)^{\frac{1}{1-\gamma}} .$$

Table 2 summarizes the results for an investor with relative risk aversion $\gamma = 3$ that have been computed by running Monte Carlo simulations. Technically, we simulate 250,000 paths of the economy under the full filtration and compute the perceived Brownian motions and Poisson processes defined in Section 2. Then, we implement the portfolio strategies according to the resulting sample paths of the filtered probabilities.[23] To assess the utility loss from not filtering at all, we implement a strategy with constant portfolio weights. We choose the constant weights that ex-ante yield the highest indirect utility among all constant strategies. Therefore, the corresponding utility loss provides a lower bound on the loss of an investor that implements some constant strategy. Notice that choosing constant portfolio weights is equivalent to not updating the probability of being in the calm state.

We find several relevant results reported in Table 2: First, comparing columns (a) and (b) with column (c) shows that filtering matters. Second, disregarding diffusive information always leads to negligible utility losses.[24] Third, disregarding contagious jumps causes up to six times higher utility losses, although the absolute losses are moderate. Notice however that we assume an investment horizon of 20 years. Numerical experiments not reported here show that the utility losses are almost linearly increasing in the time horizon. In a life-cycle context with a horizon of more than 50 years, losses can thus amount to about 1%, which is considered as substantial in the literature (see Cocco, Gomes, and Maenhout (2005)). On the other hand, losses are much smaller for more risk averse investors ($\gamma = 10$). This is because these investors put less money into stocks and thus misspecified strategies matter less.

[22]Notice that this loss function is different from the one used for filtering, which is a mean-square loss function. Nevertheless, we stick to this practice, since it is standard in the portfolio choice literature. We thank an anonymous referee for bringing this issue to our attention.

[23]For each sample path, we assume an initial value of 0.5 for both state variables. Robustness checks have shown that this assumption is not crucial.

[24]This conclusion assumes that we know the true coefficients of the diffusion parts of the models. We thank an anonymous referee for bringing that to our attention.

5 Conclusion

This paper studies a model for contagion effects that are triggered by certain crashes in asset prices. Since the individual cannot distinguish between ordinary crashes and crashes that let the economy slip into contagion, he filters the probability of being in the contagion state from price observations. We relate our model to frameworks using self-exciting processes and show that only a model with contagious jumps induces a self-exciting process for jump intensities. We also compare our model to a model with regime switching and show that, in general, this model class does not induce self-exciting dynamics. This is because the corresponding mean reversion levels of filtered jump intensities are higher.

Our results also show that the risk of contagion and the partial information about the current state of the economy can have a substantial effect on an investor's optimal portfolio strategy. Since the investor only learns gradually about whether the economy has entered the contagion state, he gradually adjusts his portfolio towards the portfolio that would be optimal in the (unobservable) contagion state. This causes him to underreact to jumps that induce contagion and to overreact to ordinary jumps. On the other hand, agents using a regime switching filter implement a noisier portfolio strategy. In a simulation study, we evaluate the performance of several investment strategies. We find that filtering matters for portfolio decisions. The utility losses of using the wrong filter are moderate, but can become significant if the investment horizon is large (such as 50 years in a life-cycle setting).

A Filtering the State of the Markov Chain

We derive the filter equations both in the regime switching model and in the model with contagious jumps for an economy with n risky assets. Section A.3 gives the filter of an investor who – suboptimally – neglects the information from diffusive noise and relies on jump observations only in an economy with contagious jumps.

A.1 Filtering in a Model with Contagious Jumps

Under the full filtration $\mathcal{F}$, the asset prices follow

$$dS_{i,t}/S_{i,t-} = \mu_i^{p_t} dt + v_i d\widetilde{W}_{i,t} - L_i \sum_{j=1}^{3} dN_{i,t}^j$$

where, in general, the diffusion processes $\widetilde{W}_i$ need not be mutually independent, but can be correlated. In order to keep the notations simple, we will replace the $\widetilde{W}_i$ by mutually independent Wiener processes W_i and introduce the correlation structure through the variance-covariance matrix $\Sigma\Sigma^T$, so that the full vector of asset prices follows the process

$$\begin{pmatrix} dS_{1,t}/S_{1,t-} \\ \vdots \\ dS_{n,t}/S_{n,t-} \end{pmatrix} = \begin{pmatrix} \mu_1^{p_t} \\ \vdots \\ \mu_n^{p_t} \end{pmatrix} dt + \Sigma \begin{pmatrix} dW_{1,t} \\ \vdots \\ dW_{n,t} \end{pmatrix} - \begin{pmatrix} L_1 \sum_{j=1}^3 dN_{1,t}^j \\ \vdots \\ L_n \sum_{j=1}^3 dN_{n,t}^j \end{pmatrix}.$$

Under the investor's filtration $\mathcal{G}$, the dynamics are given by

$$\begin{pmatrix} dS_{1,t}/S_{1,t-} \\ \vdots \\ dS_{n,t}/S_{n,t-} \end{pmatrix} = \begin{pmatrix} \widehat{\mu}_1 \\ \vdots \\ \widehat{\mu}_n \end{pmatrix} dt + \Sigma \begin{pmatrix} d\widehat{W}_{1,t} \\ \vdots \\ d\widehat{W}_{n,t} \end{pmatrix} - \begin{pmatrix} L_1 d\widehat{N}_{1,t} \\ \vdots \\ L_n d\widehat{N}_{n,t} \end{pmatrix}.$$

where the subjective drift and jump intensity of asset i are defined as

$$\begin{aligned} \widehat{\mu}_i &= \widehat{p}_t \mu_i^{calm} + (1 - \widehat{p}_t)\mu_i^{cont} \\ \widehat{\lambda}_i &= \widehat{p}_t \left(\lambda_i^{calm,calm} + \lambda_i^{calm,cont}\right) + (1 - \widehat{p}_t)\lambda_i^{cont,cont} \end{aligned}$$

and $\widehat{p}_t$ denotes the subjective probability of being in the calm state at time t. Note that the diffusion volatilities and correlations do not depend on the state of the economy and are known to the investor. The Brownian motions under the investor's filtration $\mathcal{G}$ satisfy

$$d\widehat{W}_t = dW_t + \Sigma^{-1}(\mu^{p_t} - \widehat{\mu})^T dt$$

and the observable jumps are driven by the processes

$$\widehat{N}_i = N_i^1 + N_i^2 + N_i^3, \qquad i \in \{A, B\}.$$

To deduce the filter equation, we build on the results of Frey and Runggaldier (2010). Our model can be viewed as a special case of theirs. The subjective probability of being in the calm state, $\widehat{p}$, can be written as

$$\widehat{p} = \frac{\sigma^{calm}}{\sigma^{cont} + \sigma^{calm}} \ .$$

The processes σ^{calm} and σ^{cont} then satisfy so-called Zakai equations. We take these Zakai equations from section 4 of Frey and Runggaldier (2010). The Zakai equations for the time between two observable jumps are given in Proposition 4.1 of their paper. Translated into their notation, our model has two states ($k = 1$: calm, $k = 2$: contagion). Our jump intensities translate into their notation as follows:

$$\begin{aligned} \overline{\lambda}(calm, y) &= \sum_{i=1}^{n} \lambda_i^{calm,calm} + \sum_{i=1}^{n} \lambda_i^{calm,cont} = \lambda^{calm,*} \\ \overline{\lambda}(cont, y) &= \sum_{i=1}^{n} \lambda_i^{cont,cont} = \lambda^{cont,*} \end{aligned}$$

are the total intensities for 'defaults' (i.e. observable jumps) in the calm and contagion state, respectively. The intensities for unobservable jumps, i.e. 'transitions' of the Markov chain without default, $\overline{q}_{k,i}^y$, are zero in our model except for $\overline{q}_{cont,calm}^y = \lambda^{cont,calm}$. Moreover, let $d\Psi_t$ be the diffusion part of the asset price which is denoted by dZ_t^n in Frey and Runggaldier (2010). Under the full filtration $\mathcal{F}$, this diffusion part reads

$$d\Psi_t \;\; = \;\; (\mu^{p_t})^T dt + \Sigma dW_t.$$

Under the investor filtration $\mathcal{G}$, this diffusion part reads

$$d\Psi_t \;\; = \;\; \widehat{\mu}^T dt + \Sigma d\widehat{W}_t \tag{18}$$

Taken together, we get the Zakai equations for the time between the $(n-1)$th and the nth observable jump

$$\begin{aligned} d\sigma_t^{calm} &= -\lambda^{calm,*}\sigma_t^{calm} dt + \lambda^{cont,calm}\sigma_t^{cont} dt + \sigma_t^{calm}\mu^{calm}\left(\Sigma\Sigma^T\right)^{-1} d\Psi_t \\ d\sigma_t^{cont} &= -\lambda^{cont,*}\sigma_t^{cont} dt - \lambda^{cont,calm}\sigma_t^{cont} dt + \sigma_t^{cont}\mu^{cont}\left(\Sigma\Sigma^T\right)^{-1} d\Psi_t. \end{aligned}$$

In contrast to Frey and Runggaldier (2010), the observable jumps in our model are not jumps to default so that the total number of assets in our economy is constant over time.[25] The continuous parts of the Zakai equations are thus independent of the number of defaults in our economy. The update of $\sigma^{Z(t)}$ in case of an observable jump ('default') is given by Corollary 4.2 and Algorithm 4.3 of Frey and Runggaldier (2010), together with the initial condition of Proposition 4.1. Taken together, the complete Zakai equations in our model are

$$\begin{aligned} d\sigma_t^{calm} &= -\lambda^{calm,*}\sigma_t^{calm}dt + \lambda^{cont,calm}\sigma_t^{cont}dt + \sigma_t^{calm}\mu^{calm}\left(\Sigma\Sigma^T\right)^{-1}d\Psi_t \\ &+ \sum_{i=1}^n \left(\frac{\lambda_i^{calm,calm}\sigma_{t-}^{calm}}{\lambda_i^{cont,cont}\sigma_{t-}^{cont} + (\lambda_i^{calm,calm}+\lambda_i^{calm,cont})\sigma_{t-}^{calm}} - \sigma_{t-}^{calm}\right) d\widehat{N}_{i,t} \quad (19) \\ d\sigma_t^{cont} &= -\lambda^{cont,*}\sigma_t^{cont}dt - \lambda^{cont,calm}\sigma_t^{cont}dt + \sigma_t^{cont}\mu^{cont}\left(\Sigma\Sigma^T\right)^{-1}d\Psi_t \\ &+ \sum_{i=1}^n \left(\frac{\lambda_i^{cont,cont}\sigma_{t-}^{cont} + \lambda_i^{calm,cont}\sigma_{t-}^{calm}}{\lambda_i^{cont,cont}\sigma_{t-}^{cont} + (\lambda_i^{calm,calm}+\lambda_i^{calm,cont})\sigma_{t-}^{calm}} - \sigma_{t-}^{cont}\right) d\widehat{N}_{i,t} \quad (20) \end{aligned}$$

Plugging (18) into (19) and (20) gives

$$\begin{aligned} d\sigma_t^{calm} &= -\lambda^{calm,*}\sigma_t^{calm}dt + \lambda^{cont,calm}\sigma_t^{cont}dt \\ &+\sigma_t^{calm}\mu^{calm}\left(\Sigma\Sigma^T\right)^{-1}\left[\frac{\sigma_t^{calm}}{\sigma_t^{cont}+\sigma_t^{calm}}(\mu^{calm})^T + \frac{\sigma_t^{cont}}{\sigma_t^{cont}+\sigma_t^{calm}}(\mu^{cont})^T\right]dt \\ &+\sigma_t^{calm}\mu^{calm}\left(\Sigma\Sigma^T\right)^{-1}\Sigma d\widehat{W}_t \\ &+\sum_{i=1}^n \left(\frac{\lambda_i^{calm,calm}\sigma_{t-}^{calm}}{\lambda_i^{cont,cont}\sigma_{t-}^{cont} + (\lambda_i^{calm,calm}+\lambda_i^{calm,cont})\sigma_{t-}^{calm}} - \sigma_{t-}^{calm}\right) d\widehat{N}_{i,t} \end{aligned}$$

and

$$\begin{aligned} d\sigma_t^{cont} &= -\lambda^{cont,*}\sigma_t^{cont}dt - \lambda^{cont,calm}\sigma_t^{cont}dt \\ &+\sigma_t^{cont}\mu^{cont}\left(\Sigma\Sigma^T\right)^{-1}\left[\frac{\sigma_t^{calm}}{\sigma_t^{cont}+\sigma_t^{calm}}(\mu^{calm})^T + \frac{\sigma_t^{cont}}{\sigma_t^{cont}+\sigma_t^{calm}}(\mu^{cont})^T\right]dt \\ &+\sigma_t^{cont}\mu^{cont}\left(\Sigma\Sigma^T\right)^{-1}\Sigma d\widehat{W}_t \\ &+\sum_{i=1}^n \left(\frac{\lambda_i^{cont,cont}\sigma_{t-}^{cont} + \lambda_i^{calm,cont}\sigma_{t-}^{calm}}{\lambda_i^{cont,cont}\sigma_{t-}^{cont} + (\lambda_i^{calm,calm}+\lambda_i^{calm,cont})\sigma_{t-}^{calm}} - \sigma_{t-}^{cont}\right) d\widehat{N}_{i,t}. \end{aligned}$$

To get the filtering equation at last, we apply Itô's Lemma to $\widehat{p} = \frac{\sigma^{calm}}{\sigma^{cont}+\sigma^{calm}}$. After some manipulations, we arrive at

$$\begin{aligned} d\widehat{p}_t &= \left((1-\widehat{p}_t)\lambda^{cont,calm} - \widehat{p}_t\sum_{i=1}^n \lambda_i^{calm,cont}\right)dt + \widehat{p}_t(1-\widehat{p}_t)(\mu^{calm}-\mu^{cont})\Sigma^{-T}d\widehat{W}_t \\ &+\sum_{i=1}^n \left(\frac{\widehat{p}_{t-}\lambda_i^{calm,calm}}{\widehat{\lambda}_i} - \widehat{p}_{t-}\right)\left(d\widehat{N}_{i,t} - \widehat{\lambda}_i dt\right). \quad (21) \end{aligned}$$

[25] The information content of the jumps is, however, the same as in their paper because of our assumption of constant jump sizes.

A.2 Filtering in a Regime Switching Model with Jumps

Next, we deduce the filter for an investor in a regime switching model with jumps. Recall the assumptions

$$\begin{aligned}\lambda_i^{calm} &= \lambda_i^{calm,calm} + \lambda_i^{calm,cont}\\ \lambda^{calm,cont} &= \lambda_A^{calm,cont} + \lambda_B^{calm,cont}\\ \lambda_i^{cont} &= \lambda_i^{cont,cont}.\end{aligned}$$

With these assumptions, the local distributions of the stock prices conditional on being in one of the two states (i.e. for $\widehat{p} = 0$ or $\widehat{p} = 1$) are the same in an economy with contagious jumps and in a regime switching economy with jumps. The only thing which changes is the fact that regime switches and stock price jumps are disentangled now.

Following a very similar logic as for the contagious filtering case, the Zakai equations in this case are:

$$\begin{aligned} d\sigma_t^{calm} &= -\sum_{i=1}^{n} \lambda_i^{calm}\sigma_t^{calm}dt + \lambda^{cont,calm}\sigma_t^{cont}dt - \lambda^{calm,cont}\sigma_t^{calm}dt \qquad (22)\\ &\quad +\sigma_t^{calm}\mu^{calm}\left(\Sigma\Sigma^T\right)^{-1}d\Psi_t + \sum_{i=1}^{n}\left(\frac{\lambda_i^{calm}\sigma_{t-}^{calm}}{\lambda_i^{cont}\sigma_{t-}^{cont} + \lambda_i^{calm}\sigma_{t-}^{calm}} - \sigma_{t-}^{calm}\right) d\widehat{N}_{i,t}\\ d\sigma_t^{cont} &= -\sum_{i=1}^{n} \lambda_i^{cont}\sigma_t^{cont}dt - \lambda^{cont,calm}\sigma_t^{cont}dt + \lambda^{calm,cont}\sigma_t^{calm}dt \qquad (23)\\ &\quad +\sigma_t^{cont}\mu^{cont}\left(\Sigma\Sigma^T\right)^{-1}d\Psi_t + \sum_{i=1}^{n}\left(\frac{\lambda_i^{cont}\sigma_{t-}^{cont}}{\lambda_i^{cont}\sigma_{t-}^{cont} + \lambda_i^{calm}\sigma_{t-}^{calm}} - \sigma_{t-}^{cont}\right) d\widehat{N}_{i,t}. \end{aligned}$$

To find these equations, one has to apply the same theorems of Frey and Runggaldier (2010) as in the previous section, adjusting the definitions slightly. Proceeding as in the previous section, the conditional expectation of being in the calm state, $\widehat{p}^{rs} = \frac{\sigma^{calm}}{\sigma^{cont}+\sigma^{calm}}$ can be determined using Itô's Lemma:

$$\begin{aligned} d\widehat{p}_t^{rs} &= \left((1-\widehat{p}_t^{rs})\lambda^{cont,calm} - \widehat{p}_t^{rs}\lambda^{calm,cont}\right)dt + \widehat{p}_t^{rs}(1-\widehat{p}_t^{rs})(\mu^{calm}-\mu^{cont})\Sigma^{-T}d\widehat{W}_t\\ &\quad + \sum_{i=1}^{n}\left(\frac{\widehat{p}_{t-}^{rs}\lambda_i^{calm}}{\widehat{\lambda}_i^{rs}} - \widehat{p}_{t-}^{rs}\right)\left(d\widehat{N}_{i,t} - \widehat{\lambda}_i^{rs}dt\right). \qquad (24)\end{aligned}$$

where $\widehat{\lambda}_i^{rs} = \widehat{p}_{t-}^{rs}\lambda_i^{calm} + (1-\widehat{p}_{t-}^{rs})\lambda_i^{cont}$.

A.3 Suboptimal Filtering in a Model with Contagious Jumps

For completeness, we also report the dynamics of the subjective probability $\widehat{p}^{pjf}$ under the smaller filtration $\mathcal{H}$ ('pure jump filter'), i.e. in the case where the investor – suboptimally

– neglects the diffusive information and relies on jump observations only. These dynamics can – informally – be obtained by setting $\mu_i^{calm} = \mu_i^{cont}$, i.e. by eliminating the information from the drift and diffusion of the asset prices:

$$
\begin{aligned}
d\widehat{p}_t^{pjf} &= \left((1 - \widehat{p}_t^{pjf})\lambda^{cont,calm} - \widehat{p}_t^{pjf} \sum_{i=1}^{n} \lambda_i^{calm,cont} \right) dt \\
&\quad + \sum_{i=1}^{n} \left(\frac{\widehat{p}_{t-}^{pjf} \lambda_i^{calm,calm}}{\widehat{\lambda}_i} - \widehat{p}_{t-}^{pjf} \right) \left(d\widehat{N}_{i,t} - \widehat{\lambda}_i dt \right).
\end{aligned} \tag{25}
$$

A more formal proof can also be deduced from Brémaud (1981), pp. 94ff., and is available from the authors upon request. If the investor filters from the observation of jumps only, the filter problem is equivalent to the problem of determining the current state of a Markov chain from observations of Markov chain transitions only, which is much simpler than the nonlinear filtering computation in the previous sections.

B Portfolio Optimization

The proofs of the portfolio results are also given for a general setup with n risky assets. Let $\pi = (\pi_1, \dots, \pi_n)$ denote the vector of portfolio weights.

B.1 Optimal Portfolios in a Model with Contagious Jumps

The filter equation in this case is given by equation (21). The budget constraint is

$$
\frac{dX_t}{X_{t-}} = rdt + \sum_{i=1}^{n} \pi_i \left(\frac{dS_{i,t}}{S_{i,t-}} - rdt \right) \tag{26}
$$

and the indirect utility function is denoted by $G(t, x, \widehat{p})$. The Bellman equation reads

$$
\begin{aligned}
\max_{\pi_1,\dots,\pi_n} \Big[\quad & G_t + G_x \cdot [\text{drift from (26)}] + G_p \cdot [\text{drift from (21)}] \\
& +0.5 G_{xx} \cdot [\text{squared volatility from (26)}] \\
& +0.5 G_{pp} \cdot [\text{squared volatility from (21)}] \\
& +G_{px} \cdot [\text{volatility from (26)}] \cdot [\text{volatility from (21)}] \\
& + \sum_{i=1}^{n} \left(G^{i,+} - G \right) \widehat{\lambda}_i \quad \Big] \quad = \quad 0
\end{aligned}
$$

where subscripts t, p and x denote partial derivatives. The notation $G^{i,+}$ (and similar notation hereafter) refers to the function G immediately after a jump in asset i. With the usual conjecture

$$G(t,x,\widehat{p}) = \frac{x^{1-\gamma}}{1-\gamma} f(t,\widehat{p}),$$

we get the following differential equation for f:

$$\max_{\pi_1,\ldots,\pi_n} \Bigg[\quad f \cdot \Big[(1-\gamma)r + (1-\gamma)\pi(\widehat{\mu} - r\mathbf{1})^T - 0.5\gamma(1-\gamma)\pi\Sigma\Sigma^T\pi^T - \sum_{i=1}^{n} \widehat{\lambda}_i\Big]$$
$$+ f_p \cdot \Big[(1-\gamma)\widehat{p}(1-\widehat{p})\pi(\mu^{calm} - \mu^{cont})^T$$
$$+(1-\widehat{p})\lambda^{cont,calm} - \widehat{p}\sum_{i=1}^{n}\lambda_i^{calm,cont} + \widehat{p}\sum_{i=1}^{n}\widehat{\lambda}_i - \widehat{p}\sum_{i=1}^{n}\lambda_i^{calm,calm}\Big]$$
$$+ f_{pp} \cdot 0.5\widehat{p}^2(1-\widehat{p})^2(\mu^{calm} - \mu^{cont})(\Sigma\Sigma^T)^{-1}(\mu^{calm} - \mu^{cont})^T$$
$$+ \sum_{i=1}^{n} f\left(t, \frac{\lambda_i^{calm,calm}}{\widehat{\lambda}_i}\widehat{p}\right) \cdot (1-\pi_i L_i)^{1-\gamma}\widehat{\lambda}_i \quad + f_t \quad \Bigg] \quad = \quad 0.$$

Taking derivatives with respect to π_i gives the first-order conditions:

$$f \cdot (\widehat{\mu}_i - r) - f \cdot \gamma\pi_i\Sigma\Sigma^T e_i + f_p \cdot \widehat{p}(1-\widehat{p})(\mu_i^{calm} - \mu_i^{cont})$$
$$-f\left(t, \frac{\lambda_i^{calm,calm}}{\widehat{\lambda}_i}\widehat{p}\right) \cdot L_i(1-\pi_i L_i)^{-\gamma}\widehat{\lambda}_i \quad = \quad 0.$$

This is a nonlinear system of one partial differential and n algebraic equations for the functions f and $\pi_1, \ldots, \pi_n$ with boundary conditions $f(T,\cdot) = 1$ and $f_p(T,\cdot) = 0$. Due to the nonlinear structure of the problem, we have to rely on numerical methods. We therefore only solve the special case with two risky assets using explicit finite differences. The equations for this case are stated in Proposition 3. Note that, during the algorithm, the function f has to be evaluated at points which do not exactly lie on the grid (because of the jump terms $f\left(t, \frac{\lambda_i^{calm,calm}}{\widehat{\lambda}_i}\widehat{p}\right)$. To solve this problem, we interpolate f linearly between the two nearest grid points $(t, \widehat{p}^u)$ and $\big(t, \widehat{p}^d\big)$.

B.2 Optimal Portfolios in a Regime Switching Model with Jumps

The filter equation in this case is given by (24), the budget constraint is the same as in the previous section. The drift and volatility from (21) in the Bellman equation are thus replaced by the drift and volatility of the filter (24). Applying the separation conjecture to the indirect utility function again, we arrive at the following partial differential equation:

$$
\begin{aligned}
\max_{\pi_1,\ldots,\pi_n} \Bigg[\quad & f \cdot \Big[(1-\gamma)r + (1-\gamma)\pi(\widehat{\mu} - r\mathbf{1})^T - 0.5\gamma(1-\gamma)\pi\Sigma\Sigma^T\pi^T - \sum_{i=1}^{n} \widehat{\lambda}_i^{rs}\Big] \\
& + f_p \cdot \Big[(1-\gamma)\widehat{p}^{rs}(1-\widehat{p}^{rs})\pi(\mu^{calm} - \mu^{cont})^T \\
& \qquad + (1-\widehat{p}^{rs})\lambda^{cont,calm} - \widehat{p}^{rs}\lambda^{calm,cont} + \widehat{p}^{rs}\sum_{i=1}^{n}\widehat{\lambda}_i^{rs} - \widehat{p}^{rs}\sum_{i=1}^{n}\lambda_i^{calm}\Big] \\
& + f_{pp} \cdot 0.5(\widehat{p}^{rs})^2(1-\widehat{p}^{rs})^2(\mu^{calm} - \mu^{cont})(\Sigma\Sigma^T)^{-1}(\mu^{calm} - \mu^{cont})^T \\
& + \sum_{i=1}^{n} f\left(t, \frac{\lambda_i^{calm}}{\widehat{\lambda}_i^{rs}}\widehat{p}^{rs}\right) \cdot (1-\pi_i L_i)^{1-\gamma}\widehat{\lambda}_i^{rs} \quad + f_t \quad \Bigg] \quad = \quad 0
\end{aligned}
$$

Taking derivatives with respect to π_i gives the first-order conditions

$$
\begin{aligned}
f \cdot (\widehat{\mu}_i - r) - f \cdot \gamma\pi_i\Sigma\Sigma^T e_i + f_p \cdot \widehat{p}^{rs}(1-\widehat{p}^{rs})(\mu_i^{calm} - \mu_i^{cont}) & \\
- f\left(t, \frac{\lambda_i^{calm}}{\widehat{\lambda}_i^{rs}}\widehat{p}^{rs}\right) \cdot L_i(1-\pi_i L_i)^{-\gamma}\widehat{\lambda}_i^{rs} \quad & = \quad 0.
\end{aligned}
$$

The boundary conditions are equal to those in the model with contagious jumps, and so is the numerical solution methodology by finite differences.

B.3 Optimal Portfolios with Suboptimal Filtering

If the investor uses the suboptimal pure jump filter instead of the optimal one in an economy with contagious jumps, the optimal portfolio weights can be computed similarly again. The budget constraint equals the one in the case with contagious filtering. The drift and volatility from (21) in the Bellman equation are replaced by the drift and volatility of the suboptimal filter (25). Since the filter equation (25) contains only drift terms and jump processes, the second-order partial derivatives with respect to $\widehat{p}^{pjf}$ vanish and we end up with the following partial differential equation:

$$
\begin{aligned}
\max_{\pi_A,\pi_B} \Bigg[\quad & f \cdot \Big[(1-\gamma)r + (1-\gamma)\pi(\widehat{\mu} - r\mathbf{1})^T - 0.5\gamma(1-\gamma)\pi\Sigma\Sigma^T\pi^T - \sum_{i=1}^{n}\widehat{\lambda}_i\Big] \\
& + f_p \cdot \Big[(1-\widehat{p}^{pjf})\lambda^{cont,calm} - \widehat{p}^{pjf}\sum_{i=1}^{n}\lambda_i^{calm,cont} + \widehat{p}^{pjf}\sum_{i=1}^{n}\widehat{\lambda}_i - \widehat{p}^{pjf}\sum_{i=1}^{n}\lambda_i^{calm,calm}\Big] \\
& + \sum_{i=1}^{n} f\left(t, \frac{\lambda_i^{calm,calm}}{\widehat{\lambda}_i}\widehat{p}^{pjf}\right) \cdot (1-\pi_i L_i)^{1-\gamma}\widehat{\lambda}_i + f_t \quad \Bigg] \quad = \quad 0.
\end{aligned}
\tag{27}
$$

Deriving with respect to π_i gives the first-order conditions

$$f \cdot (\widehat{\mu}_i - r) - f \cdot \gamma \pi_i \Sigma\Sigma^T e_i - f\left(t, \frac{\lambda_i^{calm,calm}}{\widehat{\lambda}_i} \widehat{p}^{pjf}\right) \cdot L_i (1 - \pi_i L_i)^{-\gamma} \widehat{\lambda}_i = 0.$$

With the same boundary conditions as for the contagious filter, this results in a system of nonlinear differential and algebraic equations again. The equations for the special case with two risky assets are stated in Proposition 4. Since the differential equation (27) is of first order, the numerical solution with finite differences has to take the existence of characteristic manifolds into account. In particular, the stability of the explicit finite difference scheme depends crucially on whether one uses forward or backward differences in $\widehat{p}^{pjf}$. We resolve this issue using so-called upwind techniques where the choice of the differencing depends on the direction of the characteristics at every grid point.

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v_i	ρ	μ_i^{calm}	μ_i^{cont}	$\lambda_i^{calm,calm}$	$\lambda_i^{calm,cont}$	$\lambda_i^{cont,cont}$	$\lambda^{cont,calm}$	L_i	ξ_i	α_i
0.15	0.30	0.09	0.14	0.375	0.125	2.50	1.00	0.05	5.00	0.25

Table 1: Benchmark Parametrization

The table reports the benchmark parametrization of our model used in Section 4. These parameters imply expected annual returns of the risky assets of 0.065 in the calm state and 0.015 in the contagion state.

	(a) pure jump filter	(b) regime switching filter	(c) no filtering at all
Benchmark parametrization			
Loss (% of initial wealth)	0.06%	0.13%	5.85%
Higher intensity of contagious jumps ($\alpha_i = 0.5$)			
Loss (% of initial wealth)	0.06%	0.36%	7.55%
Higher intensity of cont-calm transitions ($\lambda^{cont,calm} = 2$)			
Loss (% of initial wealth)	0.03%	0.15%	3.21%
Higher risk aversion ($\gamma = 10$)			
Loss (% of initial wealth)	0.02%	0.04%	1.96%
Higher diffusive correlation ($\rho = 0.6$)			
Loss (% of initial wealth)	0.03%	0.11%	4.74%

Table 2: Utility losses for different investment strategies

The table reports the percentage decrease in initial financial wealth which is necessary to reduce the expected utility with the contagious filter to the expected utility (a) with the pure jump filter, (b) with the regime switching filter or (c) without filtering. In order to assess the loss from strategy (c), we implement a strategy with constant portfolio weights where the constant portfolio weights are chosen such that the strategy is ex-ante optimal. The percentage loss in column (c) is thus a lower bound for the utility loss without filtering. The benchmark parametrization is given in Table 1. The results are computed in a Monte Carlo simulation with 250,000 sample paths, a planning horizon of 20 years, and a relative risk aversion of $\gamma = 3$.

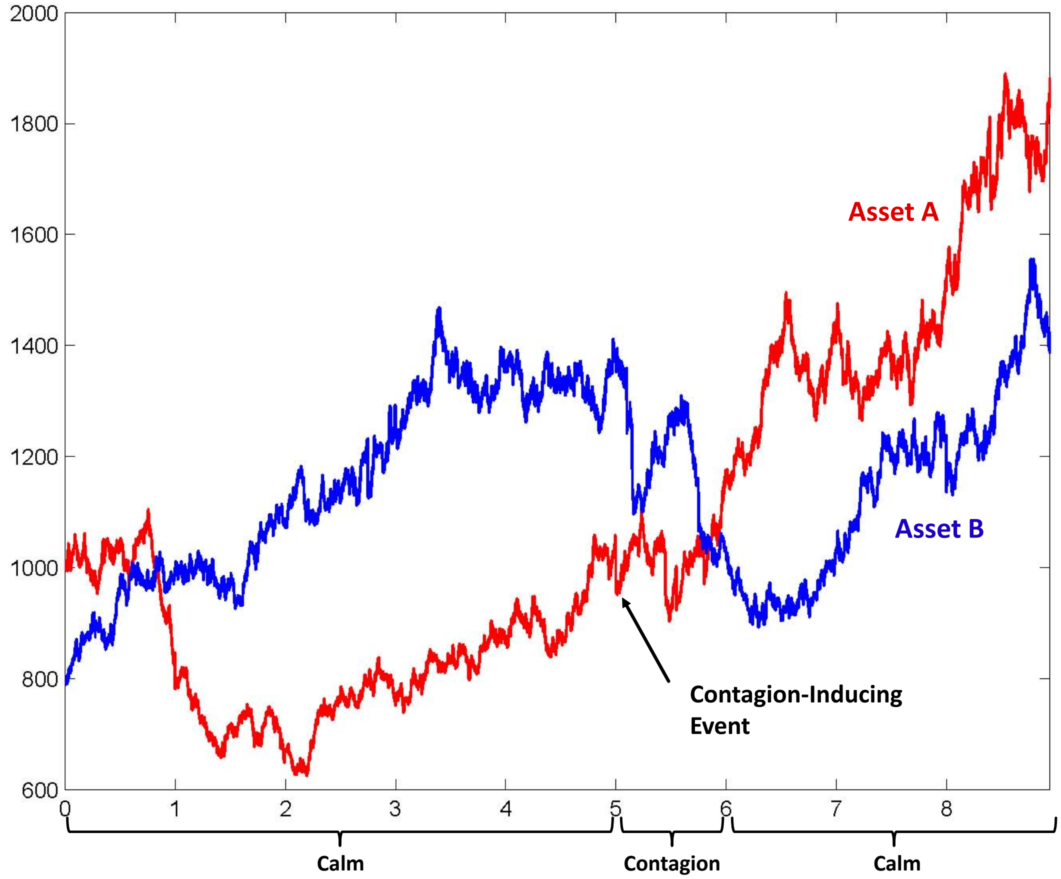

Figure 1: Typical sample paths

The figure depicts typical sample paths of the asset prices in a model with contagious jumps. A downward jump in asset A after 5 years triggers contagion. The jump probabilities for both assets are significantly larger until the economy leaves the contagion state in $t = 6$. While there is a loss in asset A as the economy enters the contagion state, the transition back to the calm state has no direct effect on the asset prices.

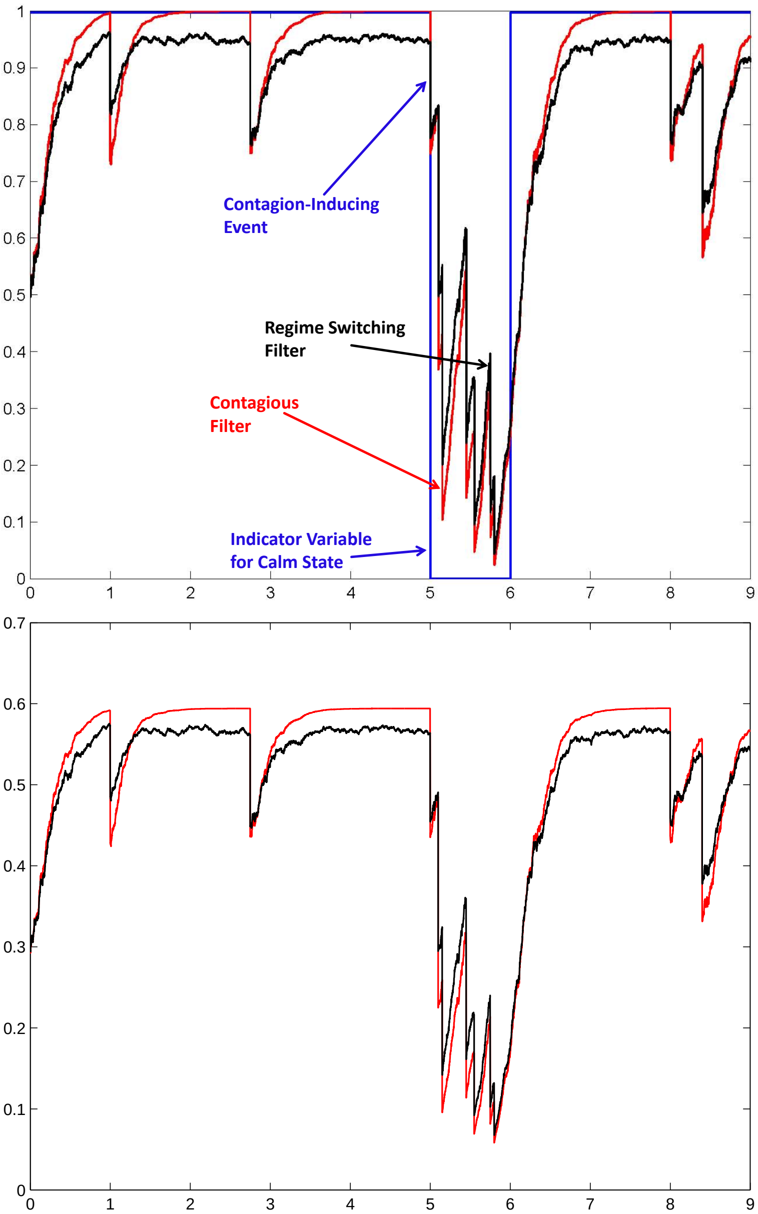

Figure 2: Sample paths of the filtered probability and the resulting portfolio weights

For the sample paths given in Figure 1, this figure depicts the filtered probability of being in the good state (upper panel) and the resulting optimal portfolio weights (lower panel) of an investor relying on the regime switching filter or the contagious filter. The filtered probabilities are adjusted downwards at every jump and revert to the mean-reversion level afterwards as long as no further jump occurs. The probability using a model with contagious jumps is given by the red path, the probability using the regime switching filter is given by the black path. The lower panel gives the optimal portfolio weights for the risky asset A. The red path depicts the optimal weights if the investor uses the contagious filter, the black path depicts the optimal portfolio for an investor using the regime switching filter. The parameters for this case are given in Table 1.

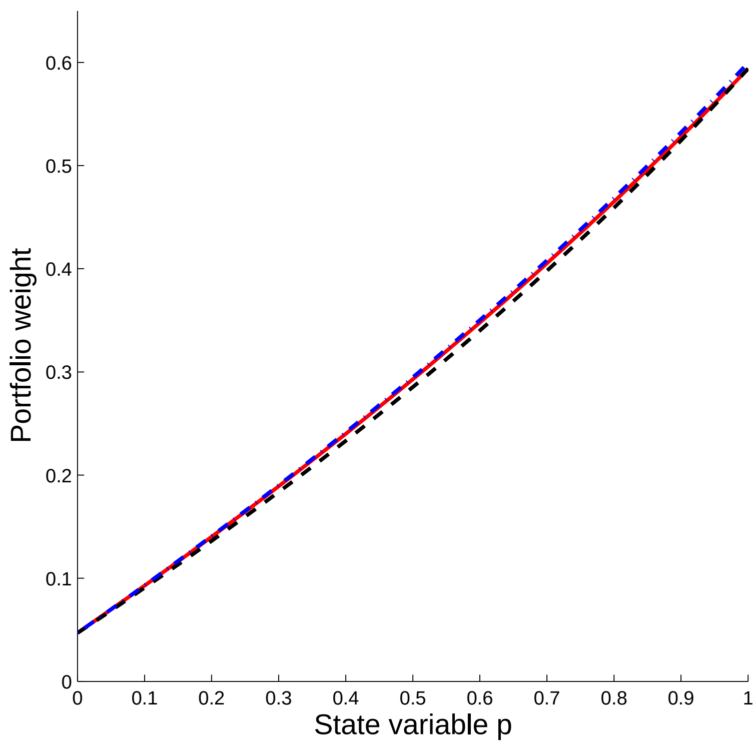

Figure 3: Optimal portfolio weights

The figure depicts the optimal portfolio weights of the risky asset A in the benchmark case. The solid red line gives the portfolio weights of an investor using the contagious filter. The dash-dotted blue line depicts the portfolio weights of an investor using the regime switching filter. The dashed black line gives the portfolio weights of an investor who uses the pure jump filter in a model with contagious jumps. Note that the optimal portfolio weights of both assets are equal since the assets are identically parameterized.

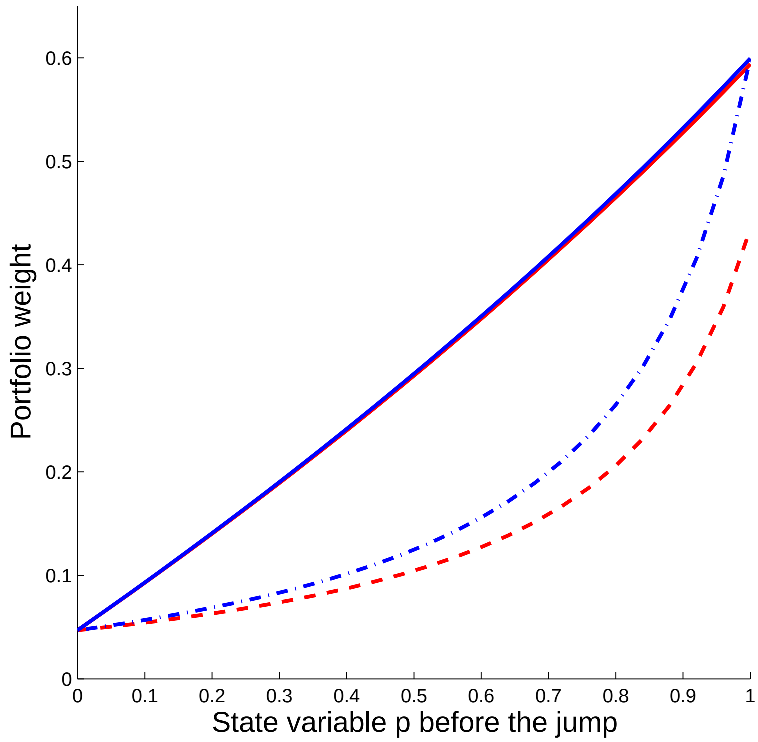

Figure 4: Portfolio update upon a jump

The figure depicts the update in the portfolio weight of the risky asset A after a jump in asset A. The solid lines show the portfolio weights of asset A before a jump as a function of the filtered probability $\widehat{p}$. The dashed lines depict the portfolio weights after a jump in asset A as a function of the filtered probability $\widehat{p}$ before that particular jump. The portfolio update is thus given by the vertical distance between the dashed and the solid line (red for the model with contagious jumps, blue for the regime switching model). The parameters can be found in Table 1.

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